

Design of Machines and Mechanical Systems (PC-BTM711)

Session 22A

Module 4: Multiplate Clutches, Energy Considerations

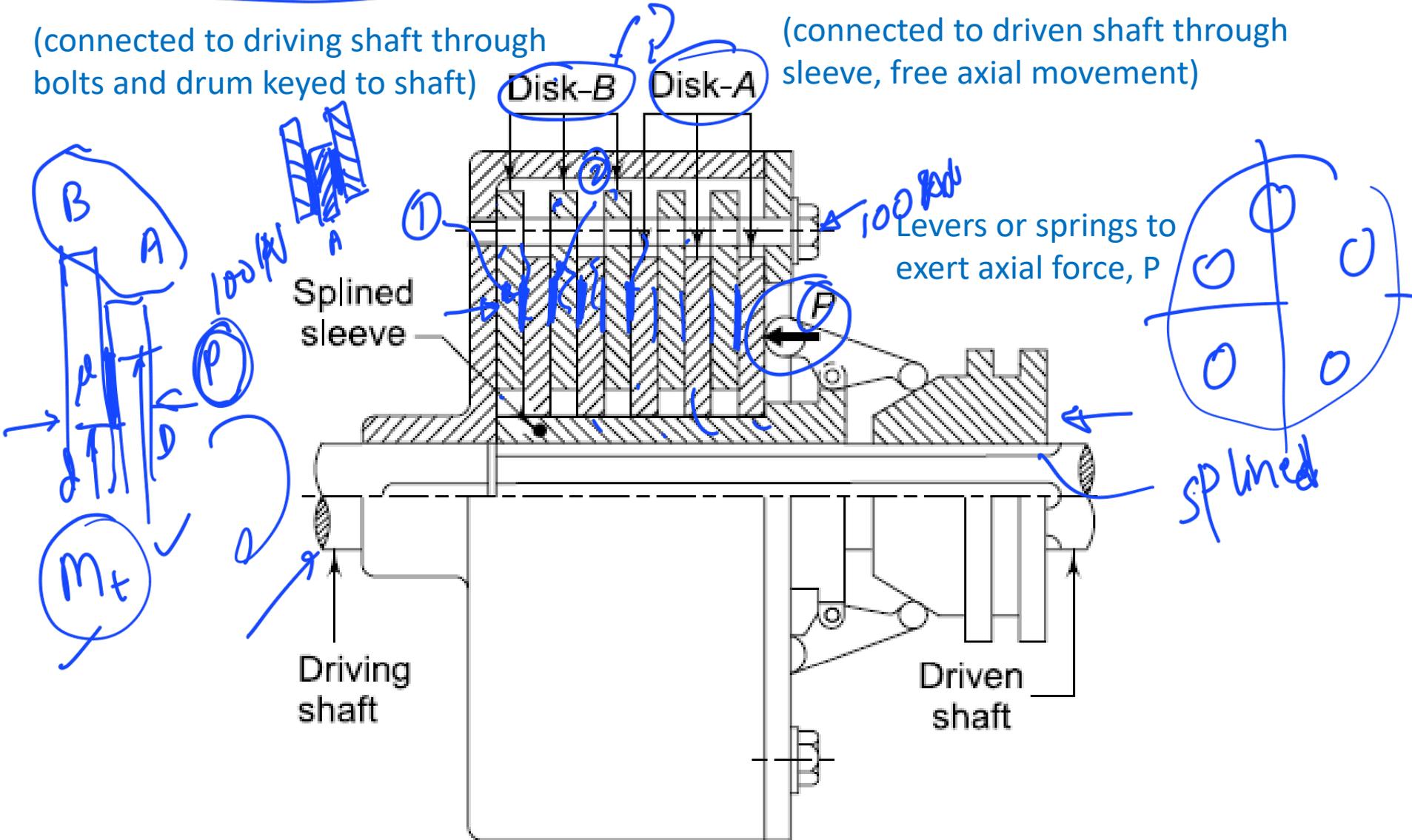
Session Outcomes

- Design of multi-disk clutches
- Develop energy equation for clutch
- Perform thermal calculations for clutch

Multi-plate Clutch

(connected to driving shaft through bolts and drum keyed to shaft)

(connected to driven shaft through sleeve, free axial movement)



Comparisons

- Single-disk vs Multi-disk clutch
- Dry vs Wet clutch

Dry - Single-plate

- ① 1 pair
For same torque capacity
- ② Size: Large, Radial ↑
- ③ Heat generation: Same
- ④ Temp: ~~Higher~~, Lower
- ⑤ μ Higher

Multiplate (wet-clutch)

- ① Multiple pairs
- ② Size: Compact
- ③ Same
- ④ Higher
- ⑤ $\mu = \text{lowers}$

Dry	Wet
① we <u>need</u> <u>sealing</u> to prevent entry of foreign particles	② <u>No</u> <u>special</u> seals for foreign particles
② Less smooth	② Smoother engagement
③ <u>wear rate</u> ↑	↓

Example 2: Multi-disk clutch

An oil immersed multi-disk clutch, with friction lining on one side and steel disks on the other, is used in an application.

The torque transmitted by clutch is 100 Nm. The coefficient of friction in wet condition is 0.08. The permissible intensity of pressure on friction lining is 1.0 MPa. The outer diameter of lining is 100 mm due to space limitation. The inside diameter is 0.577 times outside diameter to obtain optimum torque capacity.

Assuming uniform wear theory, calculate the required number of disks and the clamping force.

Multiplate Clutch Design

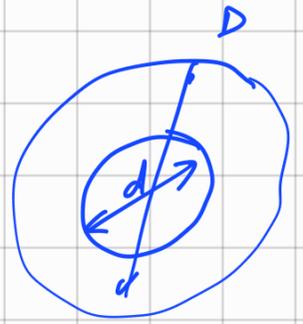
$$M_t = 100 \text{ N}\cdot\text{m} = 100,000 \text{ N}\cdot\text{mm}$$

$$p_a = 1.0 \text{ MPa}$$

$$D = 100 \text{ mm} \checkmark$$

$$\frac{d}{D} = 0.577 \Rightarrow d = 57.7 \text{ mm}$$

$$\mu = 0.08$$



Uniform Wear theory

(i) Clamping force

$$P = \frac{\pi p_a d}{2} (D - d)$$

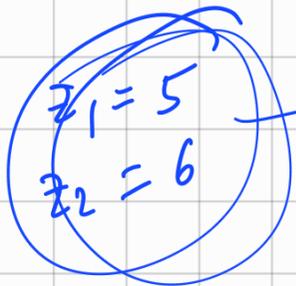
$$= \frac{\pi \times 1.0 \times 57.7}{2} (100 - 57.7) = \boxed{3,833.9 \text{ N}}$$

(ii) Required number of disks

$$M_t = \frac{\mu P Z}{4} (D + d)$$

$$\therefore Z = \frac{4 M_t}{\mu P (D + d)} = \frac{4 \times 100,000}{0.08 \times 3833.9 (100 + 57.7)}$$

$$= 8.27 \approx \boxed{9} \quad \text{or } \boxed{10}$$



$$\text{⑩} = z_1 + z_2 - 1$$

$$z = z_1 + z_2 - 1$$

odd

even

$$z_1 = z_2$$

$$z_1 = \boxed{5}$$

$$z_2 = \boxed{5}$$



QUIZ

Multi-disk Clutch

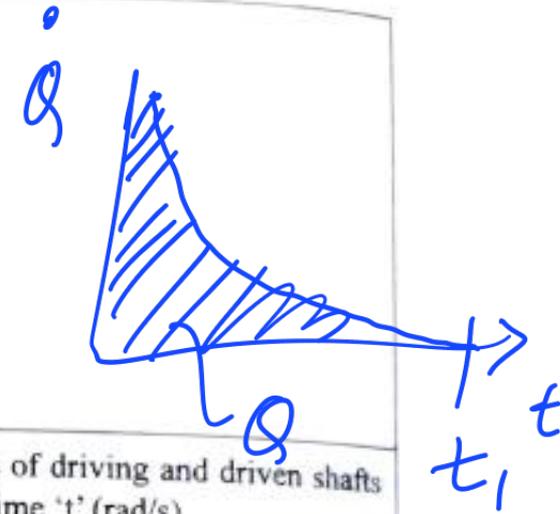
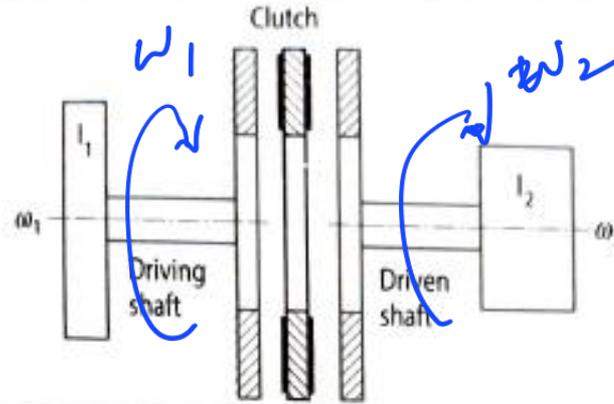
A multi-disk clutch requires 10 contact surface pairs. How many disks are required on driving and driven shaft ?

1) $z_1 = 6, z_2 = 5$

2) $z_1 = 5, z_2 = 5$

Table 11.8 Energy equations for clutches

Energy Equation



$$\omega = (\omega_1 - \omega_2) - M_t \left(\frac{I_1 + I_2}{I_1 I_2} \right) t$$

(11.29)

ω = difference in velocities of driving and driven shafts or relative velocity at time 't' (rad/s)

ω_1 = angular velocity of driving shaft (rad/s)

ω_2 = angular velocity of driven shaft (rad/s)

M_t = clutch torque (N-m)

I_1 = moment of inertia of driving shaft ($\text{kg}\cdot\text{m}^2$)

I_2 = moment of inertia of driven shaft ($\text{kg}\cdot\text{m}^2$)

t = time (s)

$$t_1 = \frac{(\omega_1 - \omega_2) I_1 I_2}{(I_1 + I_2) M_t}$$

(11.30)

t_1 = time taken by clutching operation or time at the instance when the angular velocities of two shafts become equal (s)

$$u = M_t \left[(\omega_1 - \omega_2) - M_t \left(\frac{I_1 + I_2}{I_1 I_2} \right) t \right]$$

(11.31)

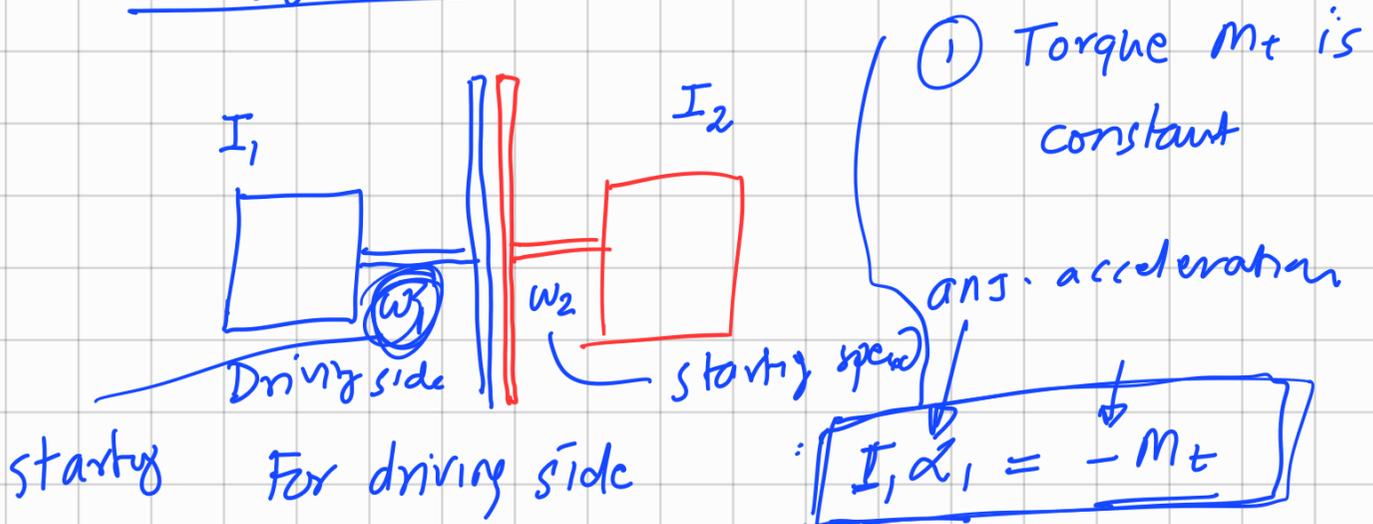
u = rate of energy dissipation (J/s)

$$E = \frac{1}{2} \frac{(\omega_1 - \omega_2)^2 I_1 I_2}{(I_1 + I_2)}$$

(11.32)

E = total energy dissipated or heat generated during clutching operation (J)

Energy Equations



$$I_1 \frac{d^2\theta_1}{dt^2} = -M_t$$

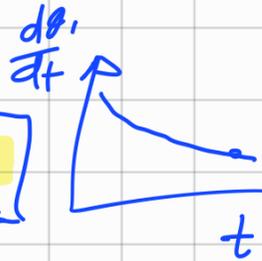
$$\therefore I_1 \frac{d\theta_1}{dt} = -M_t \cdot t + C_1$$

inst. rot. speed (driving side)

$$\text{@ } t=0, \frac{d\theta_1}{dt} = \omega_1$$

$$C_1 = I_1 \omega_1$$

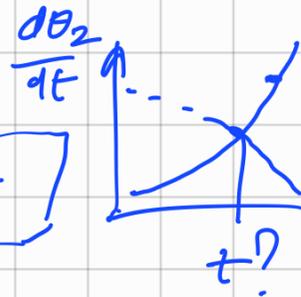
$$I_1 \frac{d\theta_1}{dt} = I_1 \omega_1 - M_t \cdot t$$



Similarly for driven side

$$I_2 \frac{d^2\theta_2}{dt^2} = +M_t$$

$$I_2 \frac{d\theta_2}{dt} = I_2 \omega_2 + M_t \cdot t$$



Relative speed, $\omega = \frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}$

$$= \left(\omega_1 - \frac{M_t \cdot t}{I_1} \right) - \left(\omega_2 + \frac{M_t \cdot t}{I_2} \right)$$

$$0 \leftarrow \omega = (\omega_1 - \omega_2) - M_t \left(\frac{1}{I_1} + \frac{1}{I_2} \right) t$$

Clutching action will be complete when, $\omega = 0$
@ $t = t_1$

$$t_1 = \frac{(\omega_1 - \omega_2) \cdot I_1 I_2}{(I_1 + I_2) m_t}$$

$\frac{\omega_1 - \omega_1}{\frac{I_1}{I_1} + \frac{I_1}{I_2}}$

Rate of heat generation, $u = m_t \cdot \omega$ ← kW

$$u = m_t \left[(\omega_1 - \omega_2) - m_t \left(\frac{I_1 + I_2}{I_1 I_2} \right) \cdot t \right]$$

Total energy dissipated from $t=0$ to $t=t_1$

$$E = \int_{t=0}^{t=t_1} u \cdot dt = \frac{1}{2} \frac{(\omega_1 - \omega_2)^2 I_1 I_2}{(I_1 + I_2)}$$

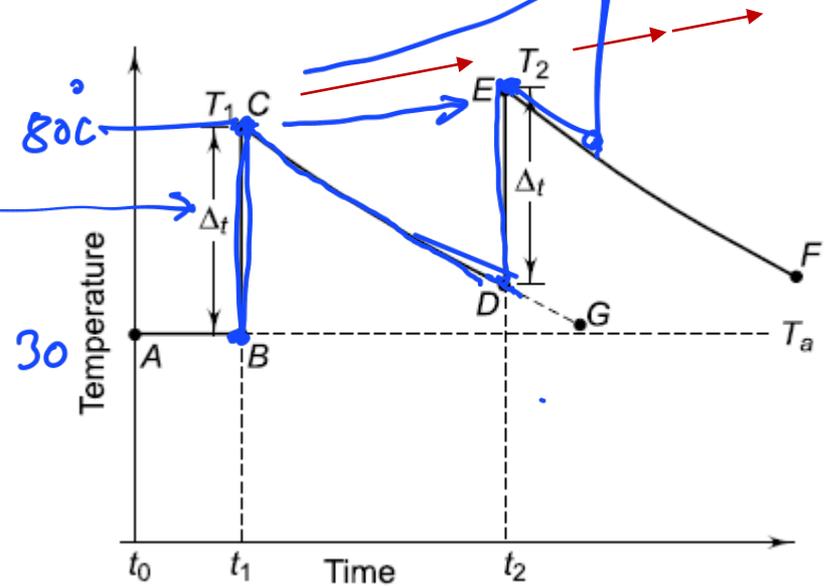
Thermal Considerations

$$Q = mc\Delta t$$

$$\Delta t = \frac{E}{mc}$$

$$T - T_a = (T_i - T_a)e^{-(Ah/mc)t}$$

Frequency of clutching is important thermal analysis parameter



Example 3: Energy considerations

A single plate clutch is designed to transmit 8 kW power at 1000 rpm.

The equivalent mass and radius of gyration of the input shaft are 10 kg and 50 mm respectively.

The equivalent mass and radius of gyration of the output shaft are 20 kg and 100 mm respectively.

Calculate:

- The time required to bring output shaft to the rated speed from rest.
- Heat generated during clutching operation

$$k_w = 8$$

$$n = 1000 \text{ rpm}$$

$$m_1 = 10 \text{ kg}, \quad k_1 = 0.050 \text{ m}$$

$$m_2 = 20 \text{ kg}, \quad k_2 = 0.10 \text{ m}$$

(i) Time for completion of clutching

$$t_c = \frac{(w_1 - w_2) \cdot I_1 I_2}{(I_1 + I_2) \cdot M_t}$$

$$w_1 = \frac{2\pi \times 1000}{60} = 104.7 \frac{\text{rad}}{\text{s}}$$

$$w_2 = 0$$

$$I_1 = m_1 k_1^2 = 0.025 \text{ kg}\cdot\text{m}^2$$

$$I_2 = m_2 k_2^2 = 0.200 \text{ kg}\cdot\text{m}^2$$

$$M_t = \frac{(60 \times 10^6) \cdot 8}{2\pi \times 1000} = 76.39 \text{ N}\cdot\text{m}$$

$$t_c = \frac{(104.7 - 0) \times 0.025 \times 0.2}{(0.025 + 0.2) \times 76.39}$$

$$= \boxed{0.0305 \text{ sec}} \rightarrow$$

(ii) Heat generated

$$E = \frac{1}{2} \frac{(w_1 - w_2)^2 I_1 I_2}{(I_1 + I_2)}$$

$$= \frac{1}{2} \times \frac{(104.7 - 0)^2 \times 0.025 \times 0.2}{(0.025 + 0.2)}$$

$$= \boxed{121.8 \text{ J}} \rightarrow$$