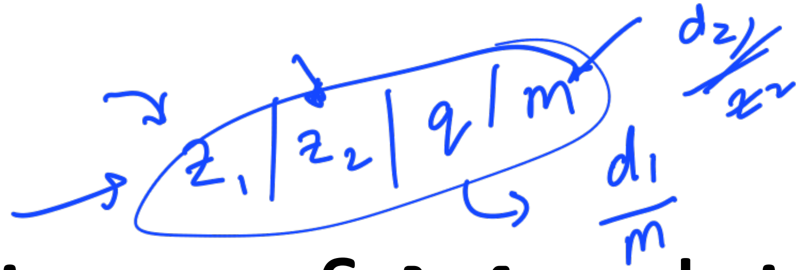
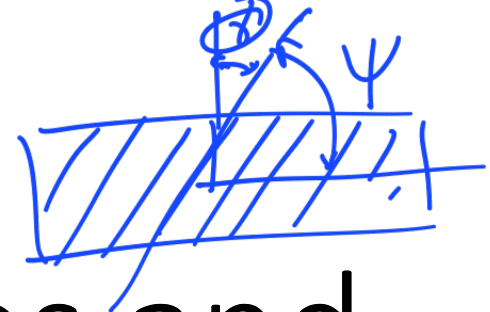




Design of Machines and Mechanical Systems (PC-BTM711)

Session 12

Module 1: Worm Gear Design

Session Outcomes

- Force analysis of worm gear
- Discuss frictional effects in worm gears
- Calculate strength rating of worm gears
- Perform calculations for thermal considerations in worm gear

QUIZ

Designation of worm gears

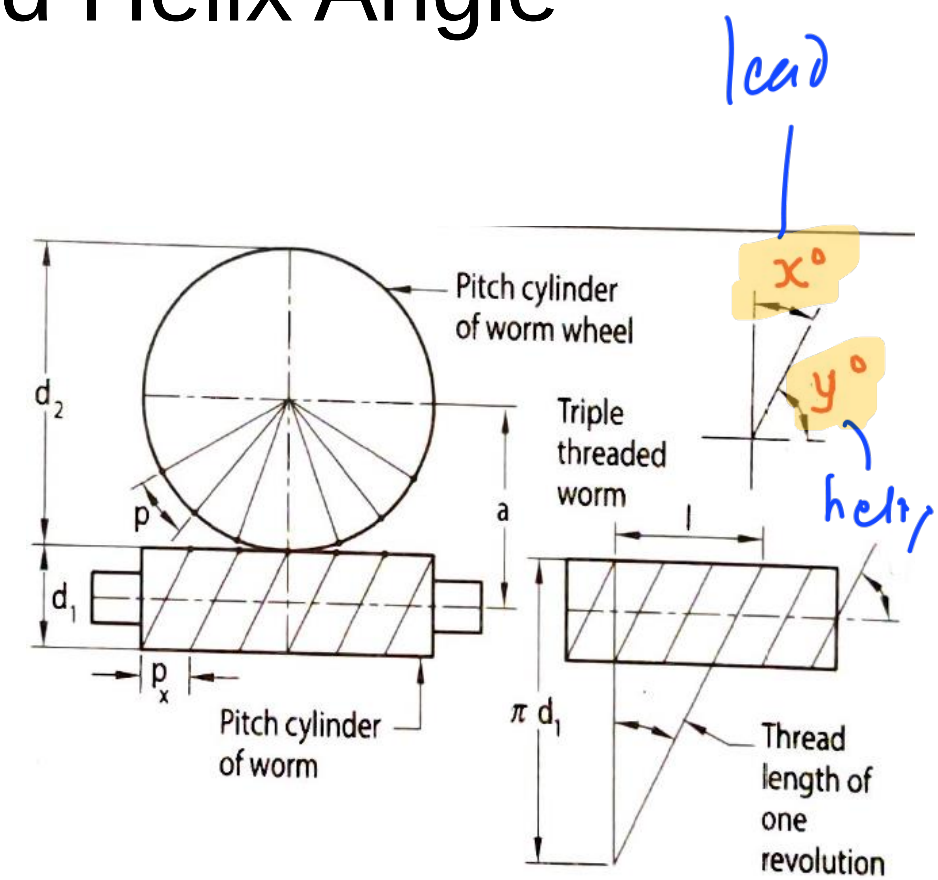
A pair worm gears is designated as 1/30/10/8. Which of the following statement is TRUE?

1. Speed ratio is 1:10
2. PCD of worm is 300 mm
3. None of the above

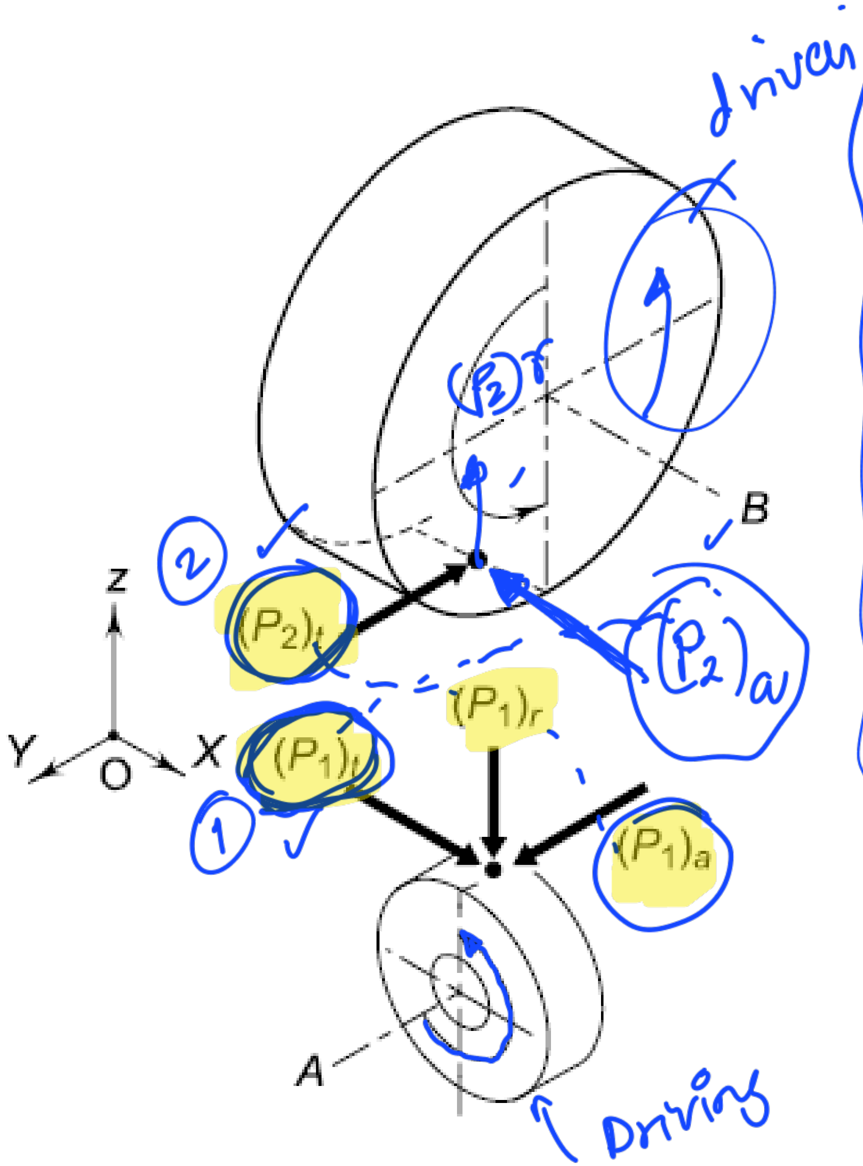
Lead and Helix Angle

The helix angle for the worm gears shown in the figure is ________

1. Angle x
2. Angle y



Force Analysis (DDB T23.10)



$$(P_1)_t = \frac{2M_t}{d_1}$$

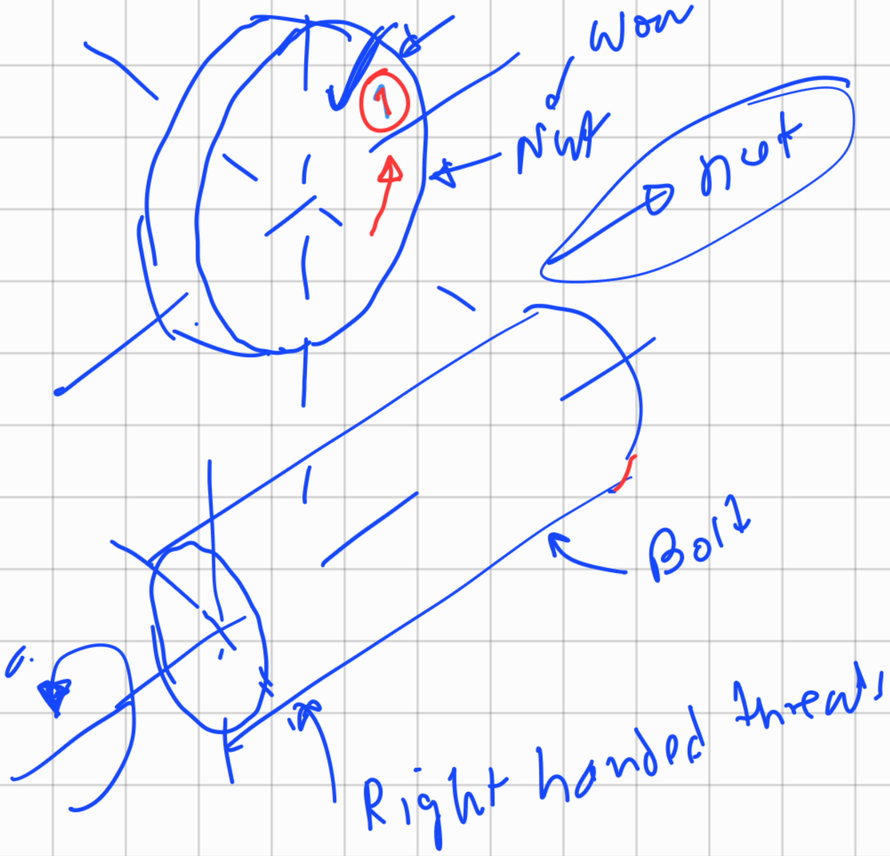
$$(P_1)_a = (P_1)_t \times \frac{(\cos\alpha\cos\gamma - \mu\sin\gamma)}{(\cos\alpha\sin\gamma + \mu\cos\gamma)}$$

$$(P_1)_r = (P_1)_t \times \frac{\sin\alpha}{(\cos\alpha\sin\gamma + \mu\cos\gamma)}$$

$$(P_2)_t = (P_1)_a$$
$$(P_2)_a = (P_1)_t$$
$$(P_2)_r = (P_1)_r$$

M_t = torque transmitted by gears (N-mm)
d_1 = pitch circle diameter of worm (mm)
γ = lead angle of worm (°)
α = normal pressure angle (°)
μ = coefficient of friction

Ref: Bhandari V.B., Design of Machine Elements (2017)

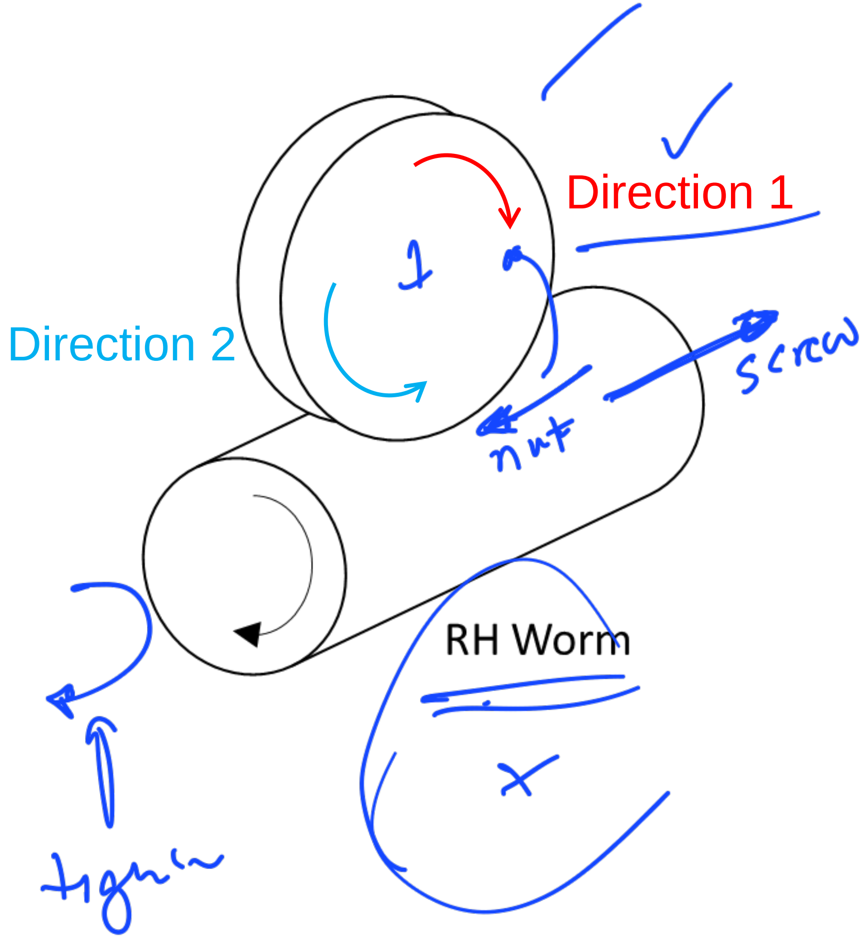

Won
Nut
nut
Bolt
Right handed threads

QUIZ

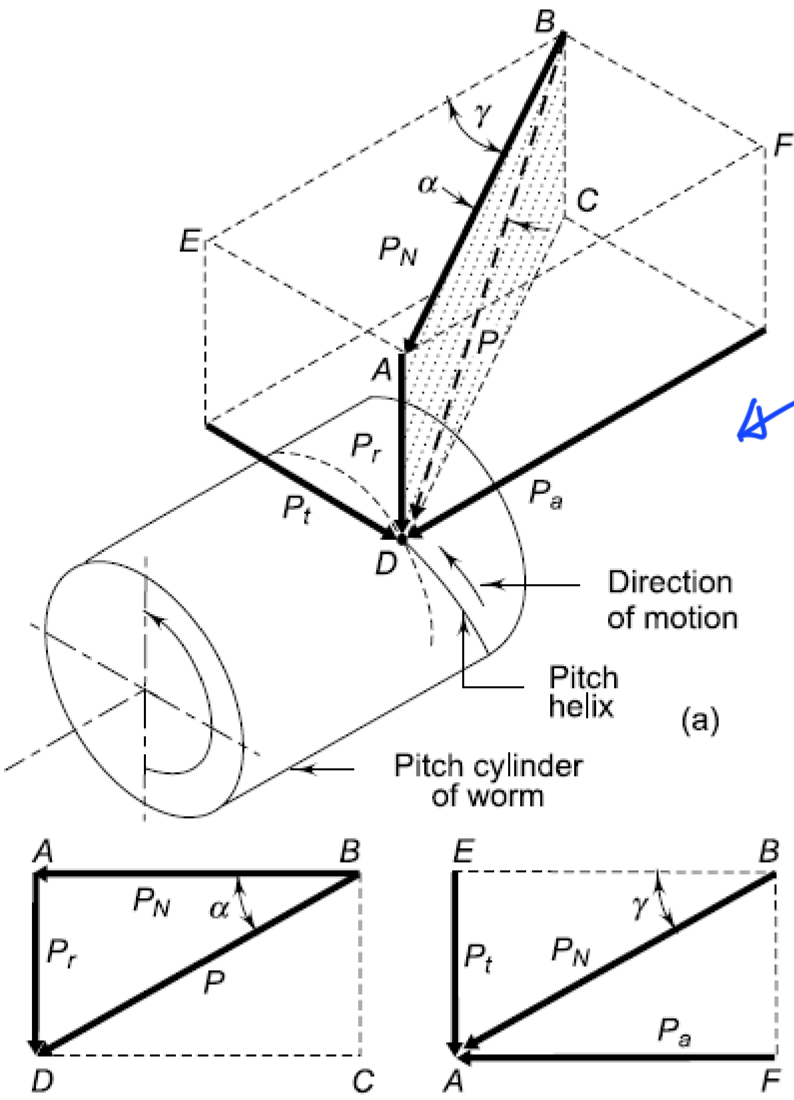
Direction of Rotation for Wheel

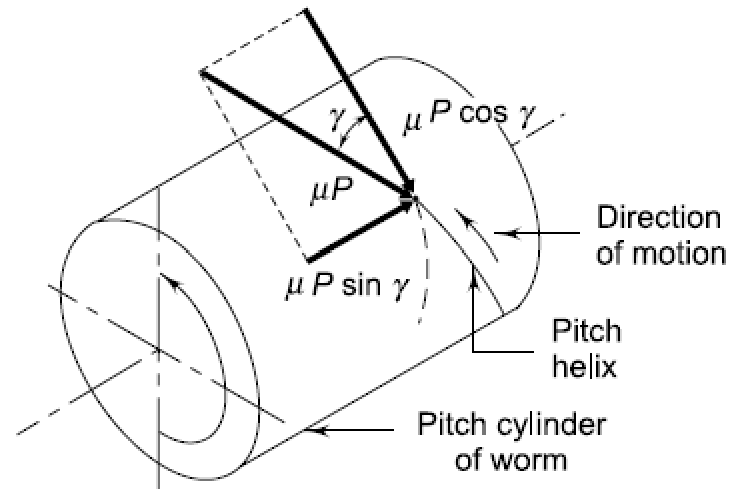
The direction of rotation of worm wheel is ________

1. Direction 1 ✓
2. Direction 2 ✗

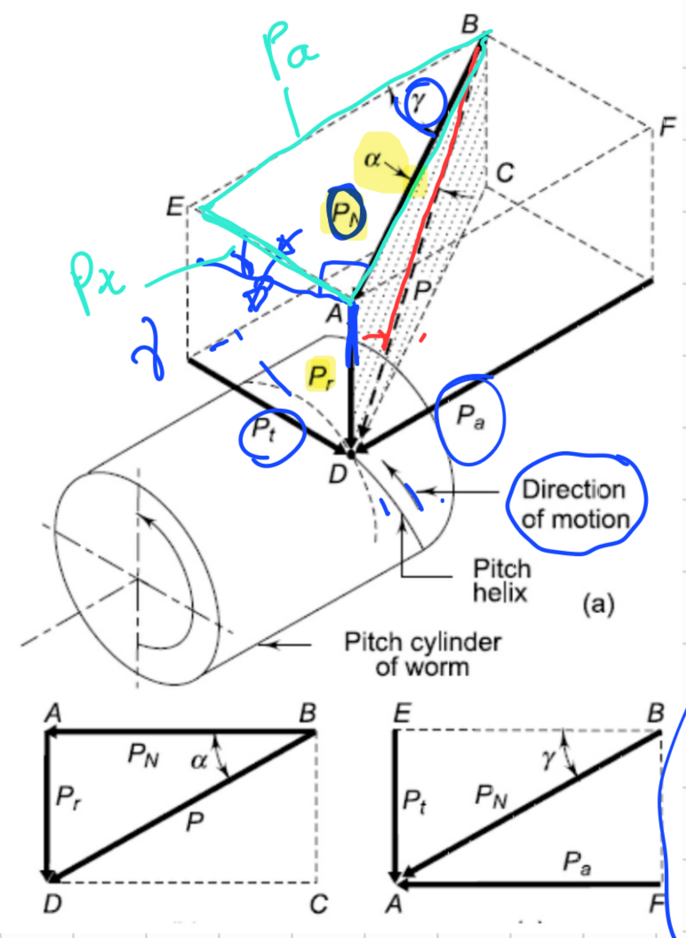


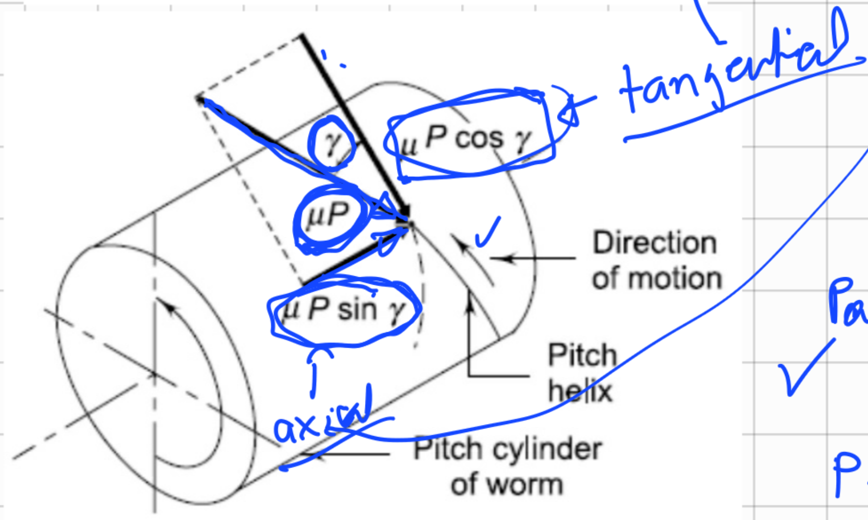
Force Analysis





Ref: Bhandari V.B., Design of Machine Elements (2017)





BD = (P) = Total force

(P_N) = horz. force

(P_r) = Radial force

γ = lead angle

$P_N = P\cos\alpha$

$P_r = P\sin\alpha$

$P_t = P_N\sin\gamma = P\cos\alpha\sin\gamma$

$P_a = P_N\cos\gamma = P\cos\alpha\cos\gamma$

$P_r = P\sin\alpha$

$P_t = P\cos\alpha\sin\gamma + \mu P\cos\gamma$

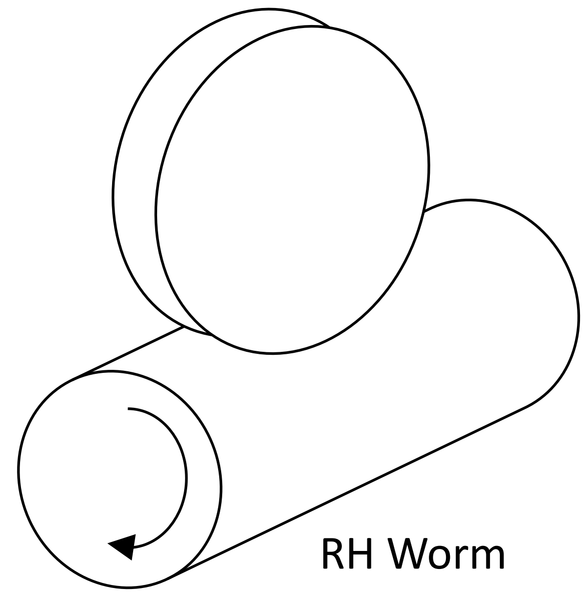
$P_a = P\cos\alpha\cos\gamma + \mu P\sin\gamma$

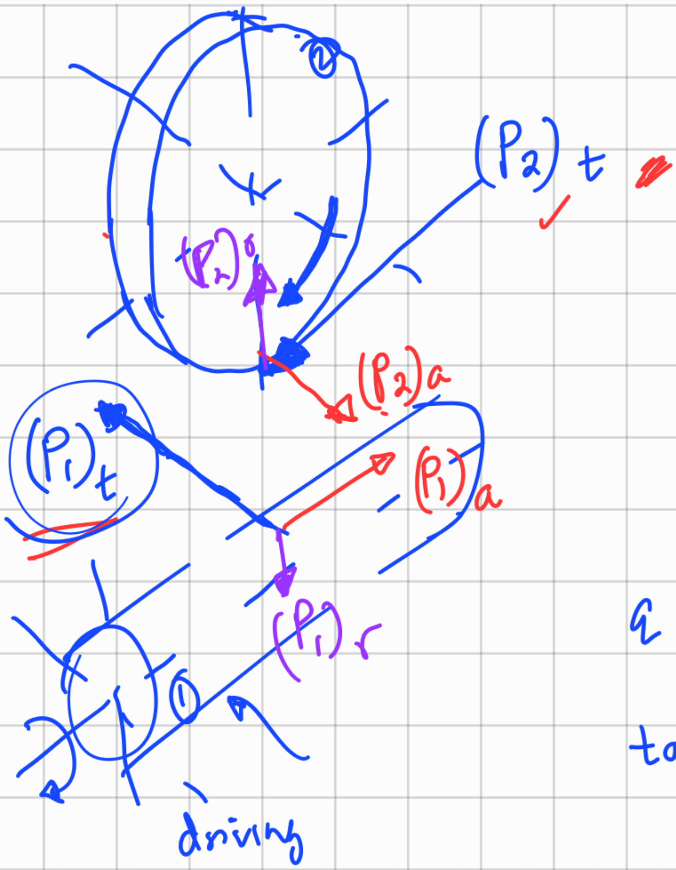
$$P_a = \left(\frac{\cos\alpha\cos\gamma - \mu\sin\gamma}{\cos\alpha\sin\gamma + \mu\cos\gamma}\right)P_t$$

$$P_r = \left(\frac{\sin\alpha}{\cos\alpha\sin\gamma + \mu\cos\gamma}\right)P_t$$

Example 1: Force Analysis

A pair of worm and worm wheel is designated as 2/40/10/4. The worm is transmitting 8 kW power at 1000 rpm to worm wheel. The coefficient of friction is 0.15 and the normal pressure angle is 20 deg. Determine the component of gear tooth force acting on the worm and worm wheel.





$kW = 8$

$n = 1000\ rpm$

$\mu = 0.15$

$z_1 = 2$

$z_2 = 40$

$q = 10$

$m = 4\ mm$

$q = \frac{d_1}{m}, \quad d_1 = qm = 10 \times 4 = \boxed{40\ mm}$

$\tan\gamma = \frac{z_1}{q} \Rightarrow \tan\gamma = \frac{2}{10} \Rightarrow \gamma = \boxed{11.31^\circ}$

$M_t = \frac{8 \times 10^6}{\left(\frac{2\pi \times 1000}{60}\right)} = 76{,}394.4\ N\text{-}mm$

$(P_1)_t = \frac{2M_t}{d_1} = 3819.7\ N$

$(P_1)_a = 3819.7 \times \frac{\cos 20 \cdot \cos 11.31 - 0.15 \sin 11.31}{\cos 20 \cdot \sin 11.31 + 0.15 \cos 11.31}$

$= 10282.2\ N$

$(P_1)_r = 3819.7 \times \frac{\sin 20}{\cos 20 \cdot \sin 11.31 + 0.15 \cos 11.31}$

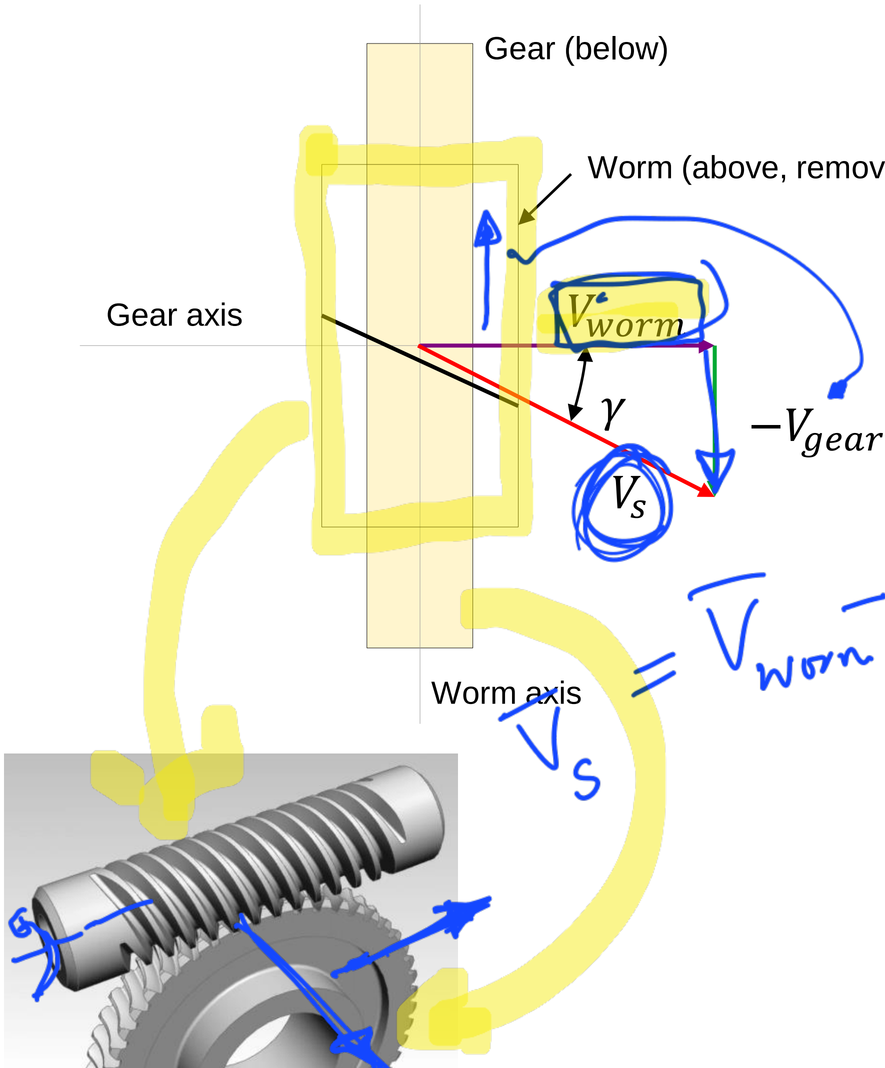
$= 3942.4\ N$

$(P_1)_t = (P_2)_a = 3819.7\ N$

$(P_1)_a = (P_2)_t = 10{,}282.2\ N$

$(P_1)_r = (P_2)_r = 3942.4\ N$

Rubbing speed, coefficient of friction and efficiency



- Rubbing speed

$$V_s = \frac{\pi d_1 n_1}{60\,000 \cos \gamma}$$

- Coefficient of friction
  - DDB T23.12
- Efficiency

$$\eta = \frac{(\cos \alpha - \mu \tan \gamma)}{(\cos \alpha + \mu \cot \gamma)}$$

- Self-locking drive

Ref: Bhandari V.B., Design of Machine Elements (2017)

$$\eta = \frac{\text{Output Power}}{\text{Input power}} \quad \{\text{worm driving}\}$$

$$= \frac{\left((P_2)_t \cdot \frac{d_2}{2} \cdot \frac{2\pi n_2}{60}\right)}{\left((P_1)_t \times \frac{d_1}{2} \times \frac{2\pi n_1}{60}\right)} \begin{matrix} \leftarrow \text{wheel} \\ \leftarrow \text{worm} \end{matrix}$$

$$\frac{(P_2)_t}{(P_1)_t} = \frac{(P_1)_a}{(P_1)_t} = \frac{\cos\alpha\cos\gamma - \mu\sin\gamma}{\cos\alpha\sin\gamma + \mu\cos\gamma}$$

$$= \frac{\cos\gamma\,(\cos\alpha - \mu\tan\gamma)}{\sin\gamma\,(\cos\alpha + \mu\cot\gamma)}$$

$$\frac{d_2}{d_1} = \frac{m z_2}{m \cdot q} = \frac{z_2}{q} = \frac{z_2/z_1}{q/z_1} = \frac{i}{1/\tan\gamma}$$

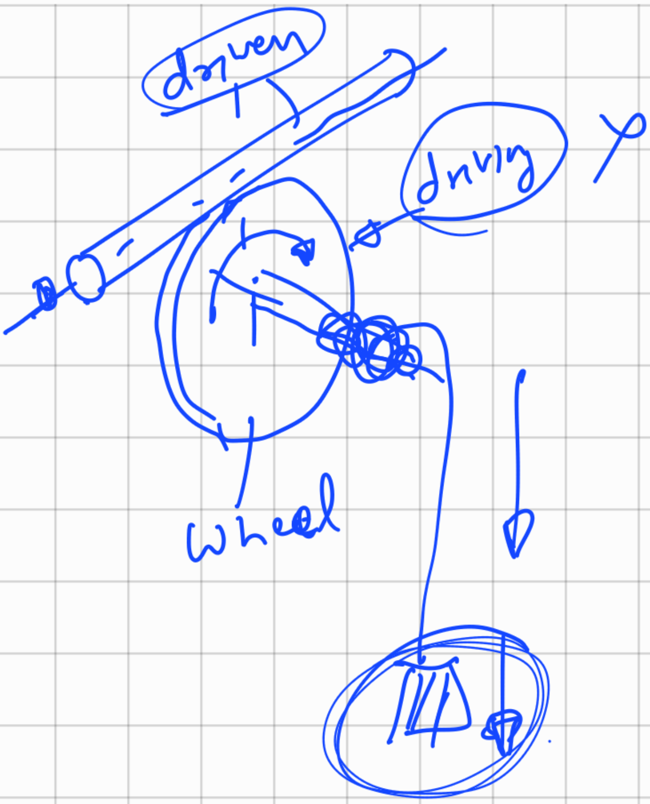
$$\frac{n_2}{n_1} = \frac{1}{i} \qquad \tan\gamma = \frac{z_1}{q}$$

$$\eta = \frac{1}{\tan\gamma} \frac{(\cos\alpha - \mu\tan\gamma)}{(\cos\alpha + \mu\cot\gamma)} (i\tan\gamma) \times \frac{1}{i}$$

$$\eta = \frac{\cos\alpha - \mu\tan\gamma}{\cos\alpha + \mu\cot\gamma} \leftarrow \text{worm is driving}$$

Self-locking

When wheel is driving



$$\eta = \frac{\cos\alpha - \mu \cot\gamma}{\cos\alpha + \mu \tan\gamma}$$

For self locking

$\eta \leq 0$

$\cos\alpha - \mu \cot\gamma \leq 0$

$\Rightarrow \mu \geq \cos\alpha \cdot \tan\gamma$

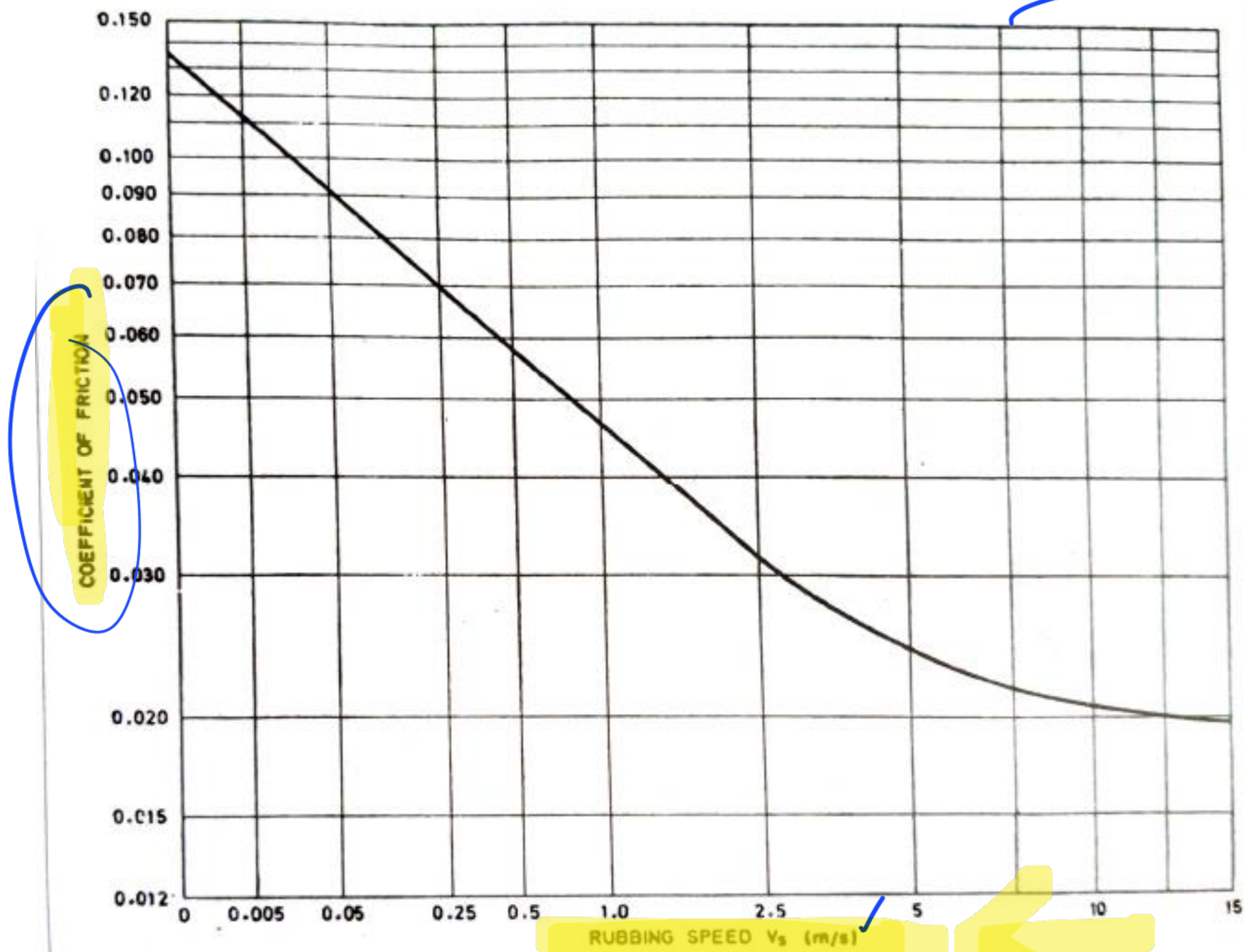
$\cos\alpha < 1$

$\sim \mu \geq \tan\gamma$

Coefficient of friction and Sliding velocity (**DDB T23.12**)

Steel # Phosphor bronze

lubrication



Ref: Bhandari V.B., Design of Machine Elements (2017)

SELF-STUDY

Selection of Material

- Worm
  - Worm threads subjected to higher number of stress cycles than worm wheel teeth (contact stress is same)
  - Worm material – case hardened steel
- Worm wheel
  - Subjected to less stress cycles
  - Tooth profile difficult to manufacture – material needs to be soft and conformable to reach final profile by plastic deformation
  - Worm wheel material – phosphor bronze
  - phosphor bronze # steel has lower coeff. of friction

Strength rating of worm gears (IS 7443)

- Max. permissible torque against bending failure (DDB T23.14), lower of the following:

$$(M_t)_1 = 17.65\, X_{b1} S_{b1} m l_r d_2 \cos\gamma$$

$$(M_t)_2 = 17.65\, X_{b2} S_{b2} m l_r d_2 \cos\gamma$$

- Max. permissible torque against wear failure (DDB T23.17), lower of the following:

$$(M_t)_3 = 18.64\, X_{c1}\, S_{c1}\, \bar{Y}_z\, (d_2)^{1.8}\, m$$

$$(M_t)_4 = 18.64\, X_{c2}\, S_{c2}\, Y_z\, (d_2)^{1.8}\, m$$

Thermal Considerations (DDB T23.21)

$$kW = \frac{k(t - t_o)A}{1000(1 - \eta)} \quad (23.37)$$

$$t = t_o + \frac{1000(1 - \eta)kW}{kA} \quad (23.38)$$

kW = power transmitted by gears on the basis of thermal consideration (kW)
k = overall heat transfer coefficient of housing walls (W/m^2 °C)
t = temperature of the lubricating oil (°C)
t_o = temperature of the surrounding air (°C)
A = effective surface area of housing (m^2)
η = efficiency of worm gears (fraction)

Fins

Heat generated = Heat dissipate

$[kW \times (1-\eta) \times 1000] = [h \cdot A \cdot \Delta t]$, $h \to k$, $\Delta t = (t - t_o)$

https://4.imimg.com/data4/DB/UI/MY-3721201/shanthi-worm-gearbox-foot-mounted-model-500x500.jpg

Example 2

It is required to design a worm gear speed reducer unit composed of worm wheel made of **sand cast and chilled phosphor bronze** and worm made of **case-hardened steel 10C4**. The center distance is **200 mm** and transmission ratio is to be **20**. The worm speed is **1500 rpm**. Design the drive and find out following.

i. Power transmitting capacity based on bending failure
ii. Power transmitting capacity based on wear failure
iii. Required effective surface area of drive, given – ambient heat transfer coefficient = 15 W/sq.m/deg.C , temperature rise of lubricating oil above surrounding temperature is to be maintained below 50 deg.C.