Design of Machines and Mechanical Systems (PC-BTM711)

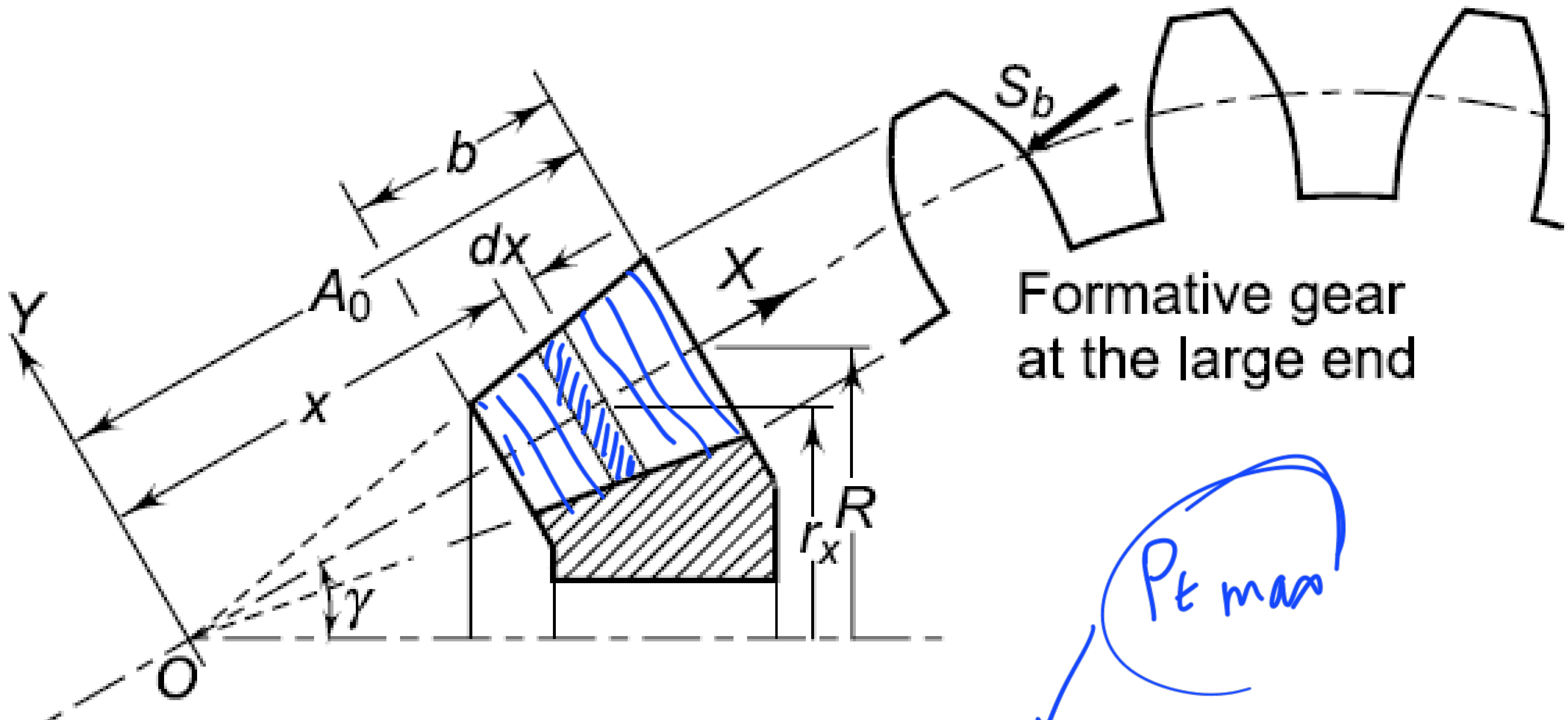
Session 11a

Module 1: Bevel Gear Strength and Wear Design

Session Outcomes

- Perform design calculations for bevel gears

Beam Strength of Bevel Gears (DDB 19.4)



$$S_b = m\,b\,\sigma_b\,Y\left[1 - \frac{b}{A_o}\right]$$

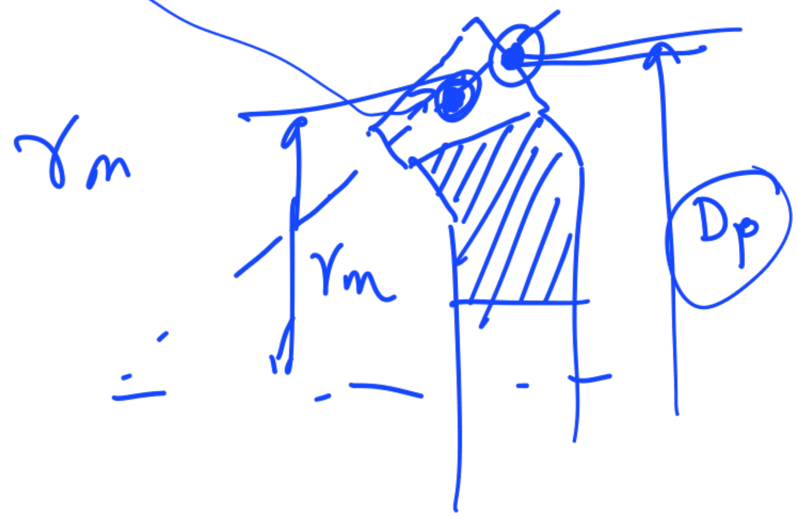
Ref: Bhandari V.B., Design of Machine Elements (2017)

QUIZ

Force Analysis of Bevel Gears

The forces on the tooth of bevel gear are calculated at _______________

1. Large end of tooth
2. Small end of tooth
3. None of the above



QUIZ

Beam Strength of Bevel Gears

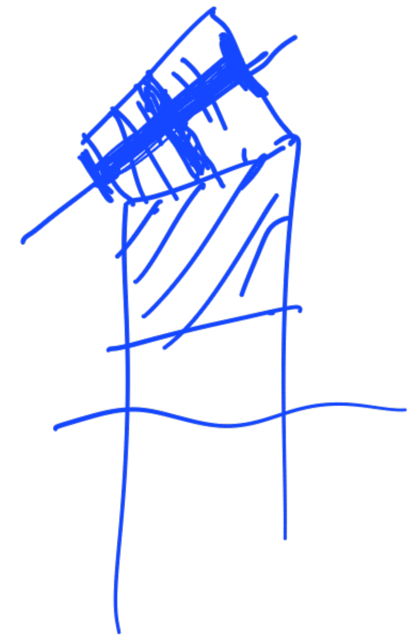
The beam strength of bevel gear is calculated at

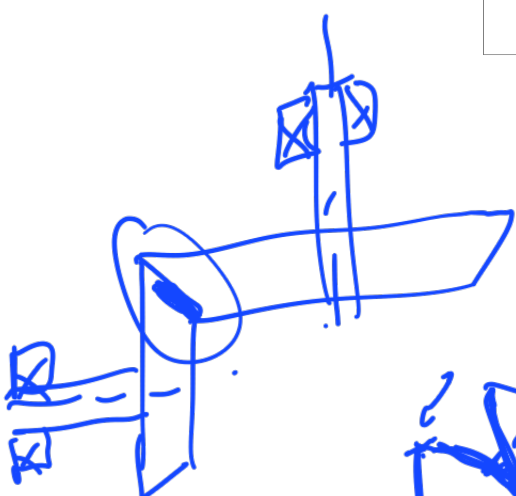
____

1. Large end of tooth ✓
2. Small end of tooth
3. None of the above

Wear Strength of Bevel Gear (DDB 19.5)

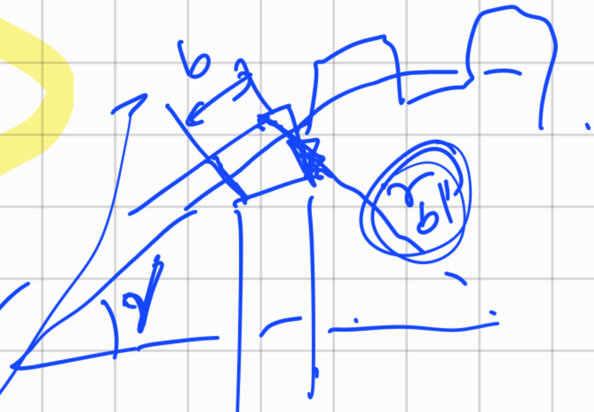
$$S_w = \frac{0.75bQD_pK}{\cos\gamma}$$





$$S_w = b\,Q\,d_p' \cdot K$$

$$d_p' = \frac{D_p}{\cos\gamma}$$



$b =$

$$Q = \frac{2z_g'}{z_g' + z_p'} \rightarrow \frac{2z_g}{z_g + z_p}$$

$$z_p' = \frac{z_p}{\cos\gamma}$$

$$z_g' = \frac{z_g}{\cos\Gamma}$$

$\sin\gamma$

$$\gamma + \Gamma = \frac{\pi}{2}$$

$$K = 0.16\left(\frac{BHN}{100}\right)^2$$

QUIZ

Wear Strength of Bevel Gears

The factor 0.75 in wear strength of bevel gears accounts for _______________

$$S_w = \frac{0.75 b Q D_p K}{\cos \gamma}$$

1. Conical profile of tooth
2. Inability to uniformly harden the entire tooth width
3. None of the above

Dynamic load

- Velocity factor (DDB 19.6)

$$C_v = \frac{6}{6+v} \quad \text{(for cut teeth)}$$

$$C_v = \frac{5.6}{5.6+\sqrt{v}} \quad \text{(for generated teeth)}$$

$P_{eff} = \frac{C_s P_t}{C_v}$

Class 1 → Commercially cut
Class 2 → cut with great care
Class 3 → ground/lapped precision gears

- Buckingham load (DDB 19.7,19.8)

$$P_d = \frac{21v(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}}$$

Table 19.8 *Maximum expected error between two meshing teeth (mm)*

Module (m) (mm)	Class – 1	Class – 2	Class – 3
Up to 4	0.050	0.025	0.0125
5	0.056	0.025	0.0125
6	0.064	0.030	0.0150
7	0.072	0.035	0.0170
8	0.080	0.038	0.0190
9	0.085	0.041	0.0205
10	0.090	0.044	0.0220

Numerical Problem

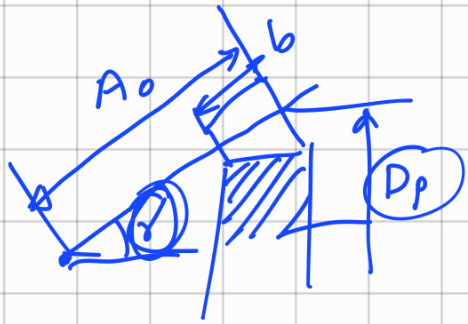
A pair of bevel gears, with 20 deg. pressure angle consists of a 24 teeth pinion meshing with 36 teeth gear. The module is 5 mm and face width is 25 mm. The material for gears is steel with UTS=800 MPa. The gear teeth are precision cut and surface hardness is 450 BHN.

The pinion rotates at 400 rpm and receives 2 kW power from an electric motor. The starting torque of motor is 150% of the rated torque. Use Buckingham's dynamic load.

Determine factor of safety against bending failure and against wear failure.

(i) Beam/bending strength calculation

$$S_b = m\, b\, \sigma_b\, Y \left[1 - \frac{b}{A_0}\right]$$



$D_p = m \cdot z_p = 5 \times 24 = 120$ mm

$D_g = m \cdot z_g = 5 \times 36 = 180$ mm

$\tan\gamma = \frac{z_p}{z_g} = \frac{24}{36} \Rightarrow \gamma = 33.69^\circ$

$z_p' = \frac{z_p}{\cos\gamma} = \boxed{28.64}$, $z_g' = \frac{z_g}{\cos\gamma} = \boxed{64.9}$

From DDB

$Y = \boxed{0.3545}$ for $z_p' =$ ~~28.64~~ 28.84

$\sigma_b = \frac{UTS}{3} = \frac{800}{3} = 266.7$ MPa

$A_0 = \frac{D_p}{2\sin\gamma} = \frac{120}{2\sin 33.69^\circ} = 108.17$ mm

$S_b = 5 \times 25 \times 266.7 \times 0.3545 \left[1 - \frac{25}{108.17}\right]$

$= \boxed{9086.8\,N}$

(ii) Wear strength

$$S_w = \frac{0.75\, b\, Q\, D_p\, K}{\cos\gamma}$$

$$Q = \frac{2 z_g'}{z_g' + z_p'} = \frac{2 \times 64.9}{64.9 + 28.84}$$

$= 1.384$

$$K = 0.16\left(\frac{450}{100}\right)^2 = 3.24 \text{ MPa}$$

$$S_w = \frac{0.75 \times 25 \times 1.384 \times 120 \times 3.24}{\cos 33.69^\circ}$$

$$= \boxed{12,125.9 \text{ N}}$$

(iii) Effective load

$$M_t = \frac{60 \times 10^6 \text{ kW}}{2\pi n} = \frac{60 \times 10^6 \times 2}{2\pi \times 400}$$

$$= \boxed{47,746.5 \text{ Nmm}}$$

Calc @ large end → $$P_t = \frac{2M_t}{D_p} = \frac{2 \times 47,746.5}{120}$$

$$= \boxed{795.8 \text{ N}}$$

$$v = \frac{2\pi n_p}{60} \times \frac{D_p}{2}$$

for m/s ↓

$$= \frac{2\pi \times 400}{60} \times \frac{120}{2} \times 10^{-3}$$

$$= 2.51 \text{ m/s}$$

$$P_d = \frac{21v\,(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}}$$

$C = 11,400$

$e = 0.0125$ mm (m = 5, class = 3 → precision cut)

$$= \frac{21 \times 2.51\,(11400 \times 0.0125 \times 25 + 795.8)}{21 \times 2.51 + \sqrt{(\;-\;\text{"}\;-\;)}}$$

$= \boxed{1934.9\ N}$

$P_{eff} = C_s P_t + P_d$

$= 1.5 \times 795.8 + 1934.9$

$= \boxed{3128.6\ N}$

(iv) FOS

$$(FOS)_{bending} = \frac{S_b}{P_{eff}} = \frac{9086.8}{3128.6}$$

$= \boxed{2.90}$

$$(FOS)_{wear} = \frac{S_w}{P_{eff}} = \frac{12,125.9}{3,128.6}$$

$$= \boxed{3.875}$$