Design of Machines and Mechanical Systems (PC-BTM711)

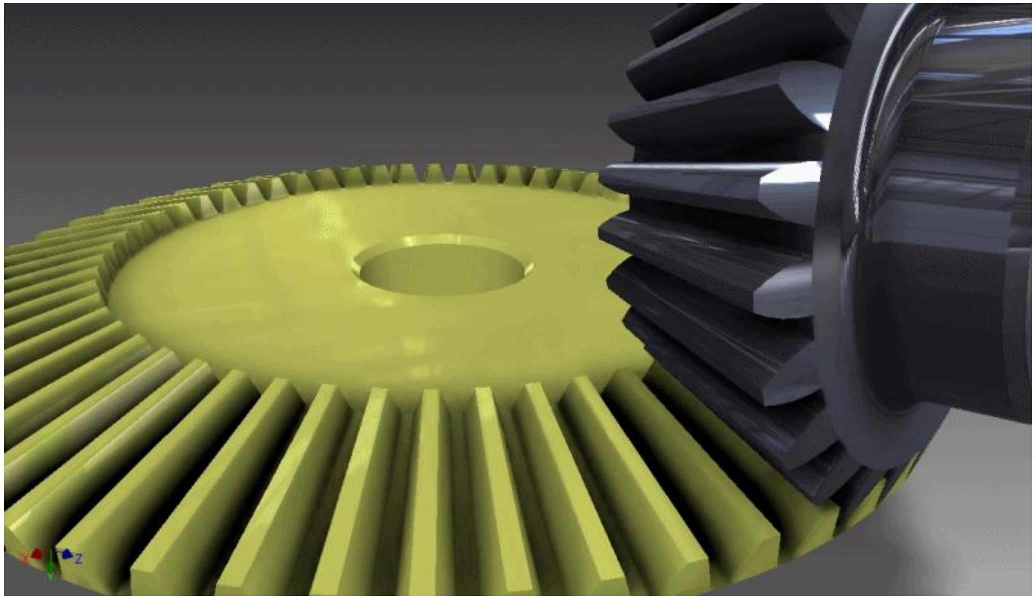
Session 10

Module 1: Bevel Gear Design

Session Outcomes

- Introduce to Bevel gears
- Analysis of forces on bevel gear tooth
- Perform design calculations for bevel gears

Bevel Gears

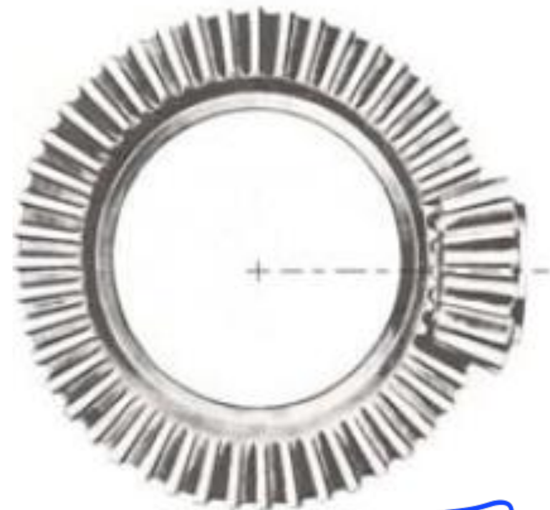


https://grabcad.com/library/bevel-gear-9

https://grabcad.com/library/helical-bevel-gears-coupling-coppia-conica-elicoidale-1

Types of Bevel Gears

Bevel Gears



Straight Bevel Gears

Spiral Bevel Gears

Hypoid Bevel Gears
(spiral with offset axes)

Credits: NPTEL course by Prof. K.Gopinath & Prof. M.M.Mayuram

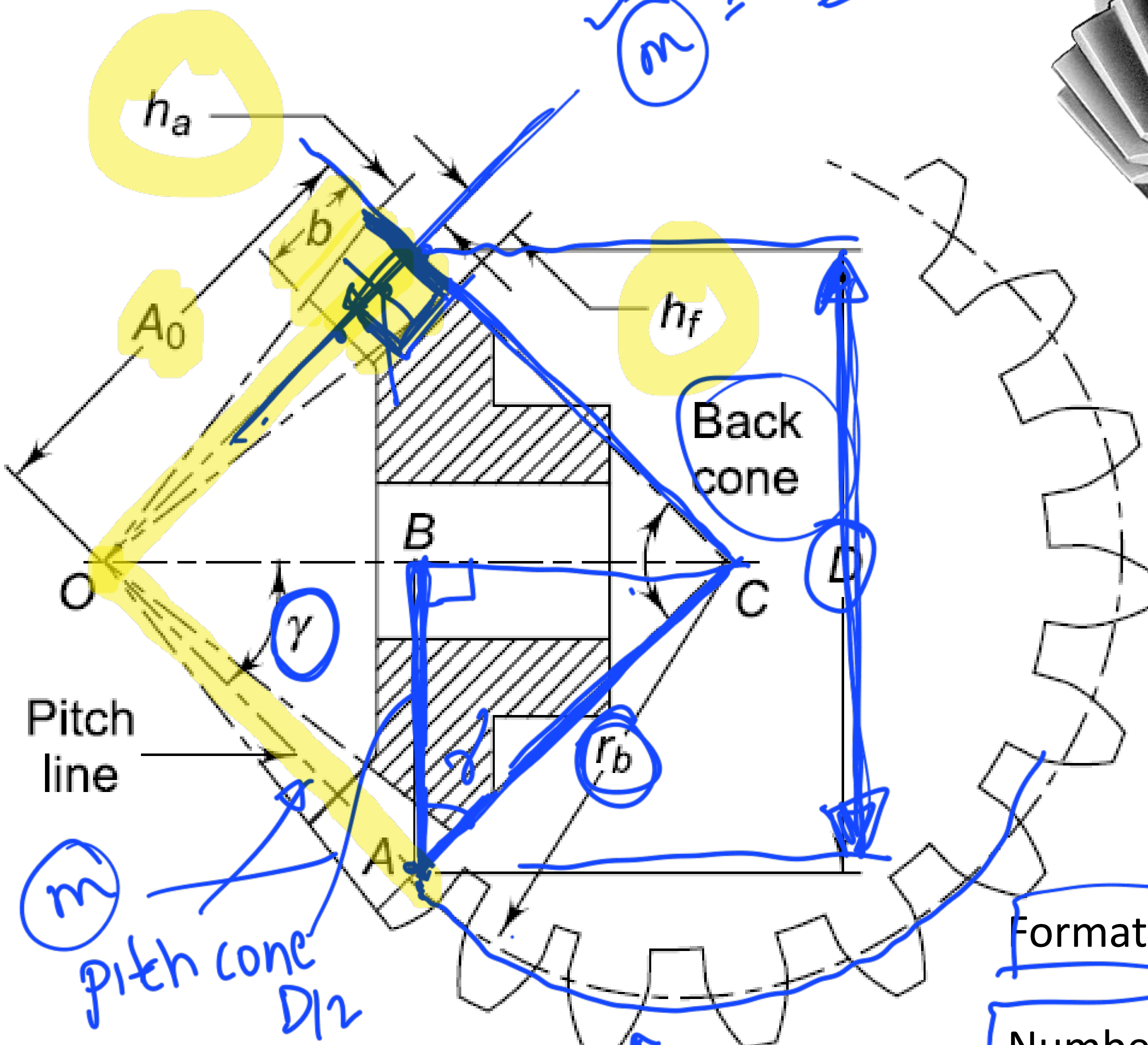
SELF-STUDY

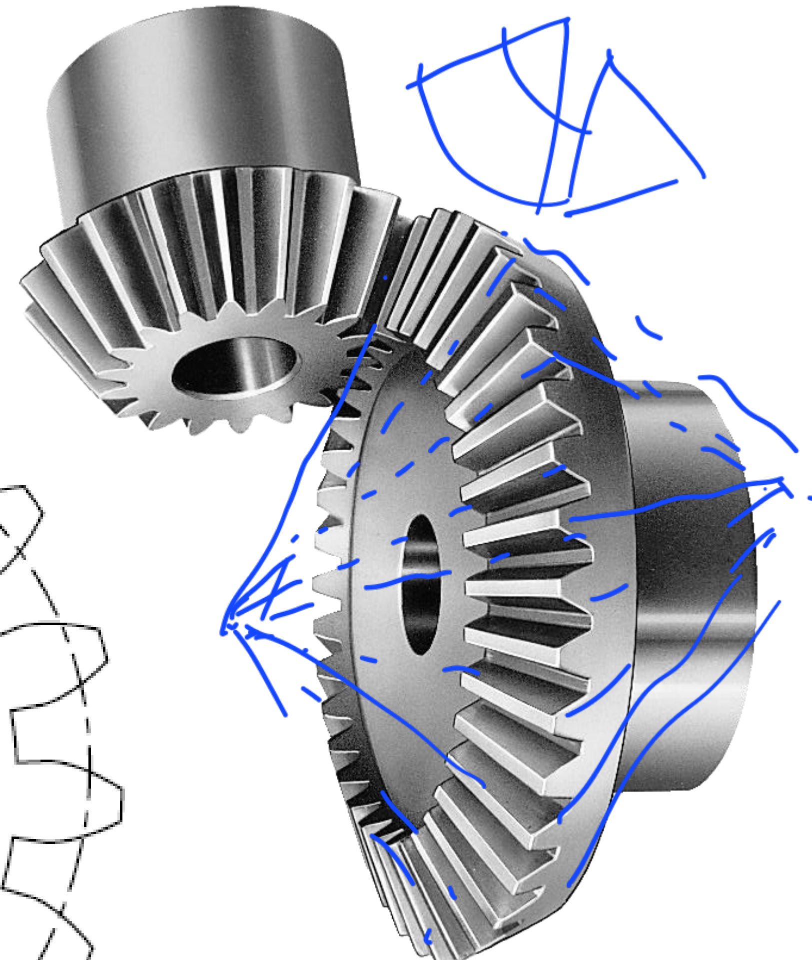
Types of Bevel Gears

- Based on tooth shape
  - Straight
  - Spiral
- Based on pitch angle
  - External bevel gear
  - Crown bevel gear
  - Internal bevel gear
- Special Categories
  - Miter gears
  - Skew bevel gears
  - Hypoid gears
  - Face gears

Ref: Bhandari V.B., Design of Machine Elements (2017)

Terminology





Formative spur gear

Number of virtual teeth

Ref: Bhandari V.B., Design of Machine Elements (2017)

$$m = \frac{D}{Z} = \frac{2 r_b}{Z'} \leftarrow ?$$

$r_b \rightsquigarrow D$

$$r_b = \frac{AB}{\cos\gamma} = \frac{D/2}{\cos\gamma}$$

$$r_b = \frac{D}{2\cos\gamma}$$

$$\frac{D}{Z} = \frac{2 \times \left(\frac{D}{2\cos\gamma}\right)}{Z'}$$

$$\boxed{Z' = \frac{Z}{\cos\gamma}}$$

QUIZ

Dimensions of Bevel Gear

The dimensions of bevel gear (module, pitch circle diameter, etc.) are always specified at _______

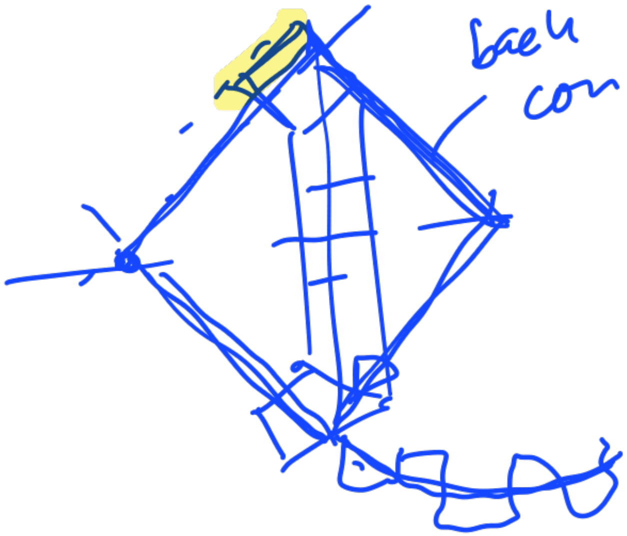
1. Large end of tooth
2. Small end of tooth
3. Mid-point of the width of tooth

QUIZ

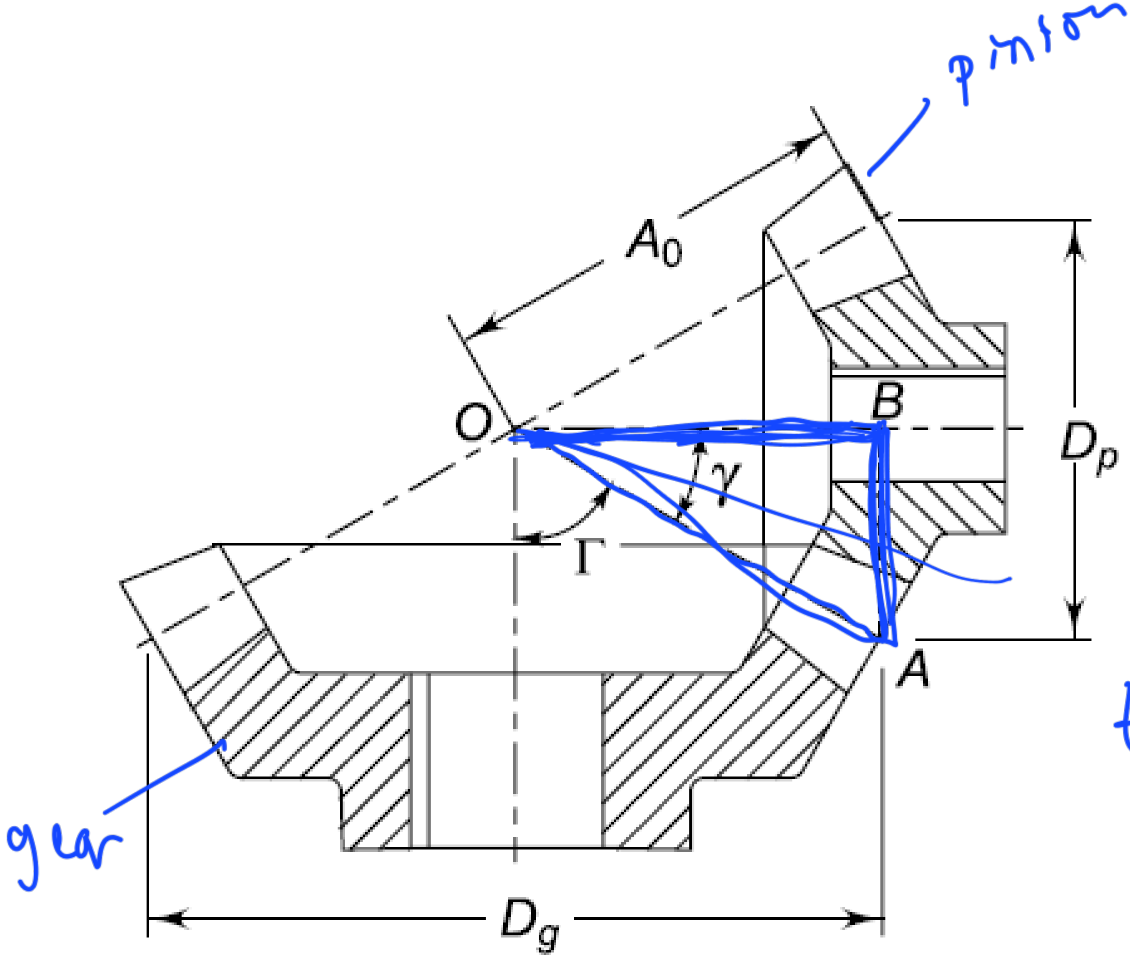
Formative Gear

The radius of formative gear is equal to ______

1. Slant distance of pitch cone
2. Slant distance of back cone
3. None of above



Pitch cone angles



$$tan\gamma = \frac{z_p}{z_g}$$

$$tan\Gamma = \frac{z_g}{z_p}$$

$$\tan\gamma = \frac{D_p/2}{D_g/2} = \frac{m z_p}{m z_g}$$

Ref: Bhandari V.B., Design of Machine Elements (2017)

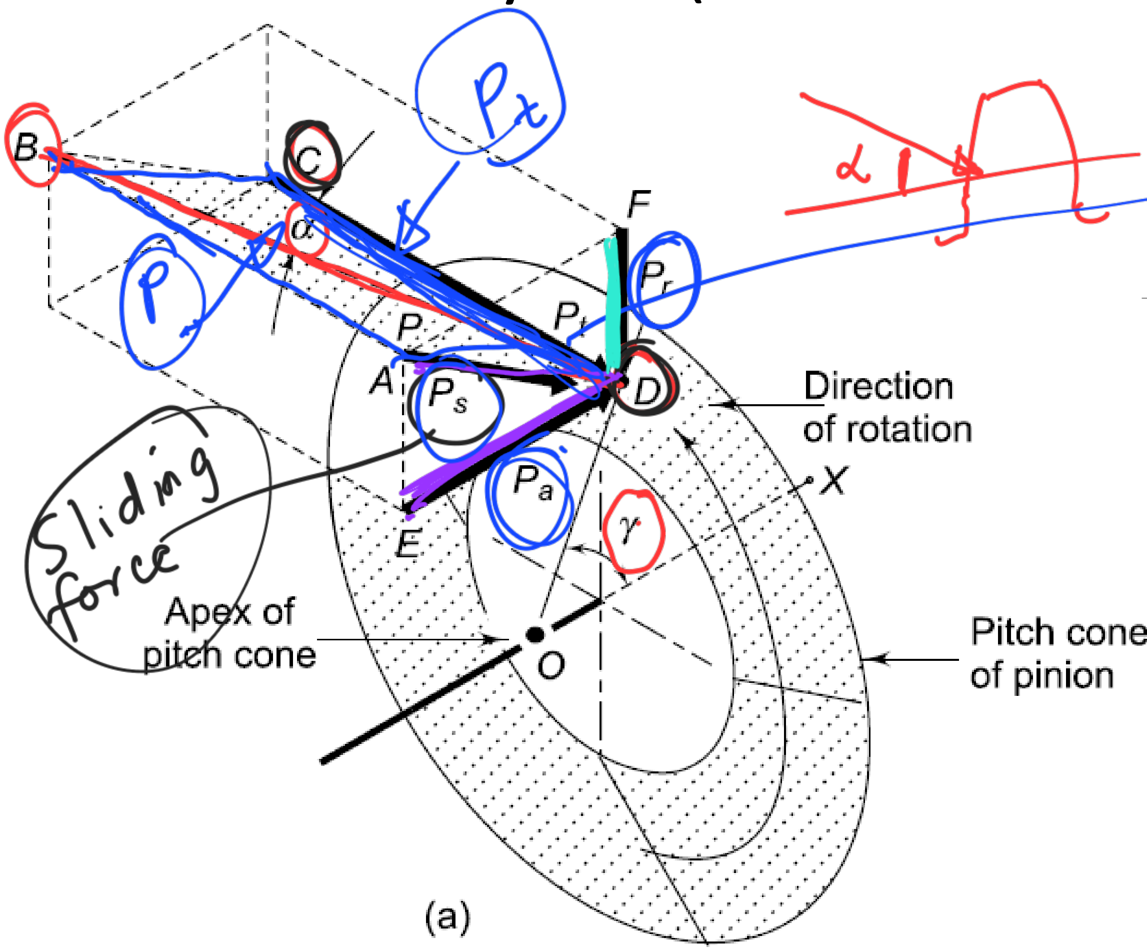
QUIZ

Pitch Cone Angle of Bevel Gear

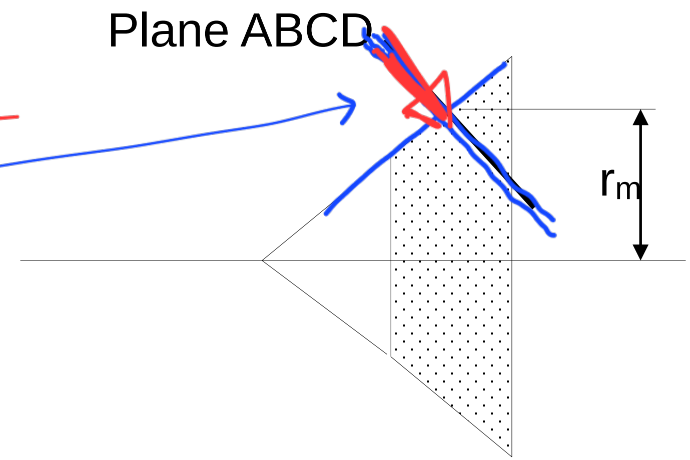
The equation $tan\gamma = \frac{z_p}{z_g}$ is always true for all bevel gear configurations.

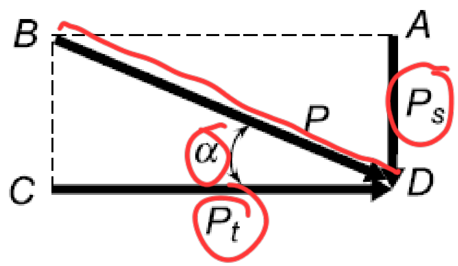
1. True
2. False ✓

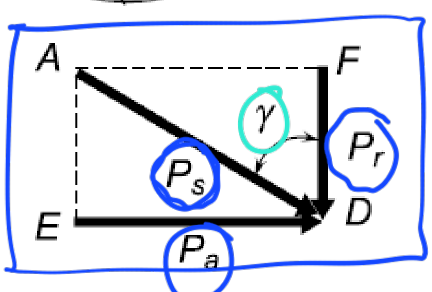
Force Analysis (calc. at mean radius)



(a)







Design Data Book T19.3

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_p}$$

$$r_m = \left[\frac{D_p}{2} - \frac{b \sin\gamma}{2}\right]$$

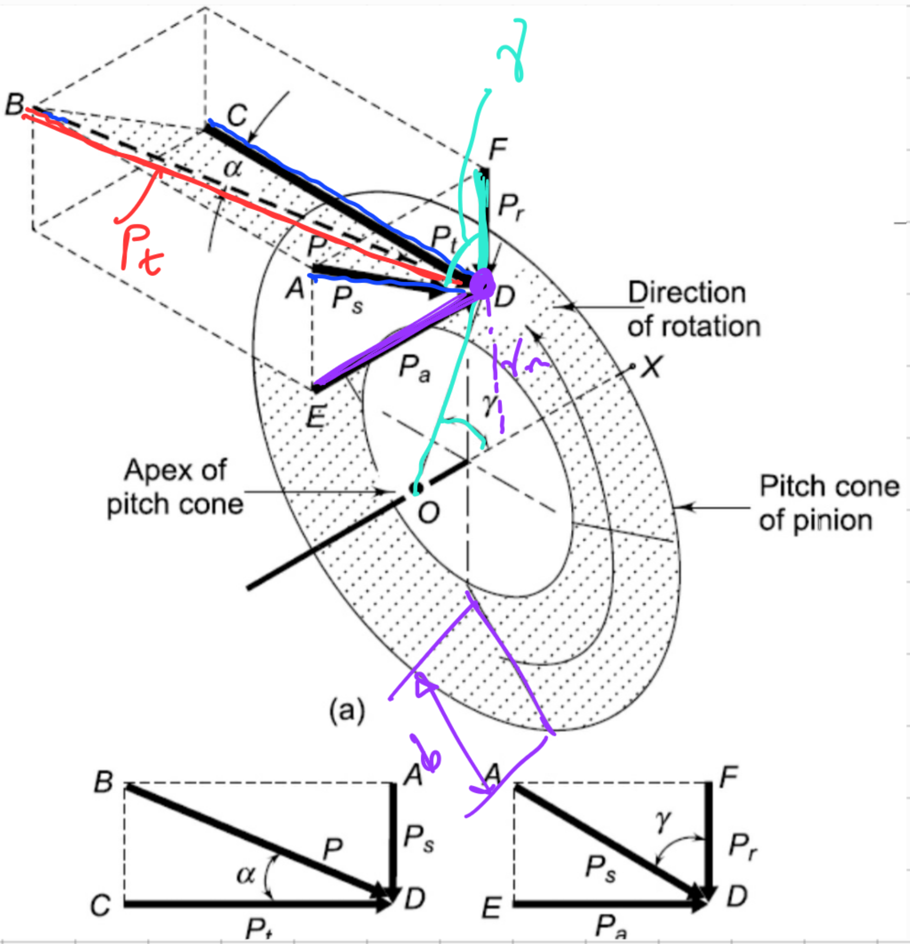
$$P_t = \frac{M_t}{r_m}$$

$$P_r = P_t \tan\alpha \cos\gamma$$

$$P_a = P_t \tan\alpha \sin\gamma$$

where suffix 'p' is used for pinion

Ref: Bhandari V.B., Design of Machine Elements (2017)



Tangential force $P_t = P\cos\alpha$

Sliding force $\rightarrow P_s = P\sin\alpha$

$\therefore P_r = P\sin\alpha \cdot \cos\gamma$

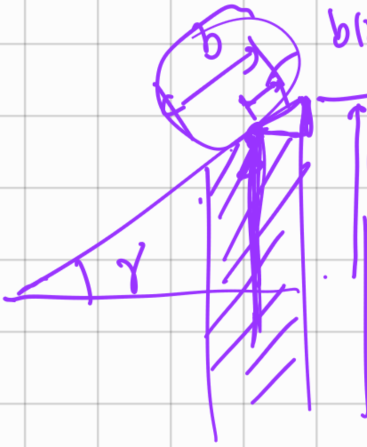
$P_a = P\sin\alpha \cdot \sin\gamma$

$$\frac{P_r}{P_t} = \frac{\sin\alpha\cos\gamma}{\cos\alpha}$$

$\Rightarrow P_r = P_t \cdot \tan\alpha \cdot \cos\gamma$

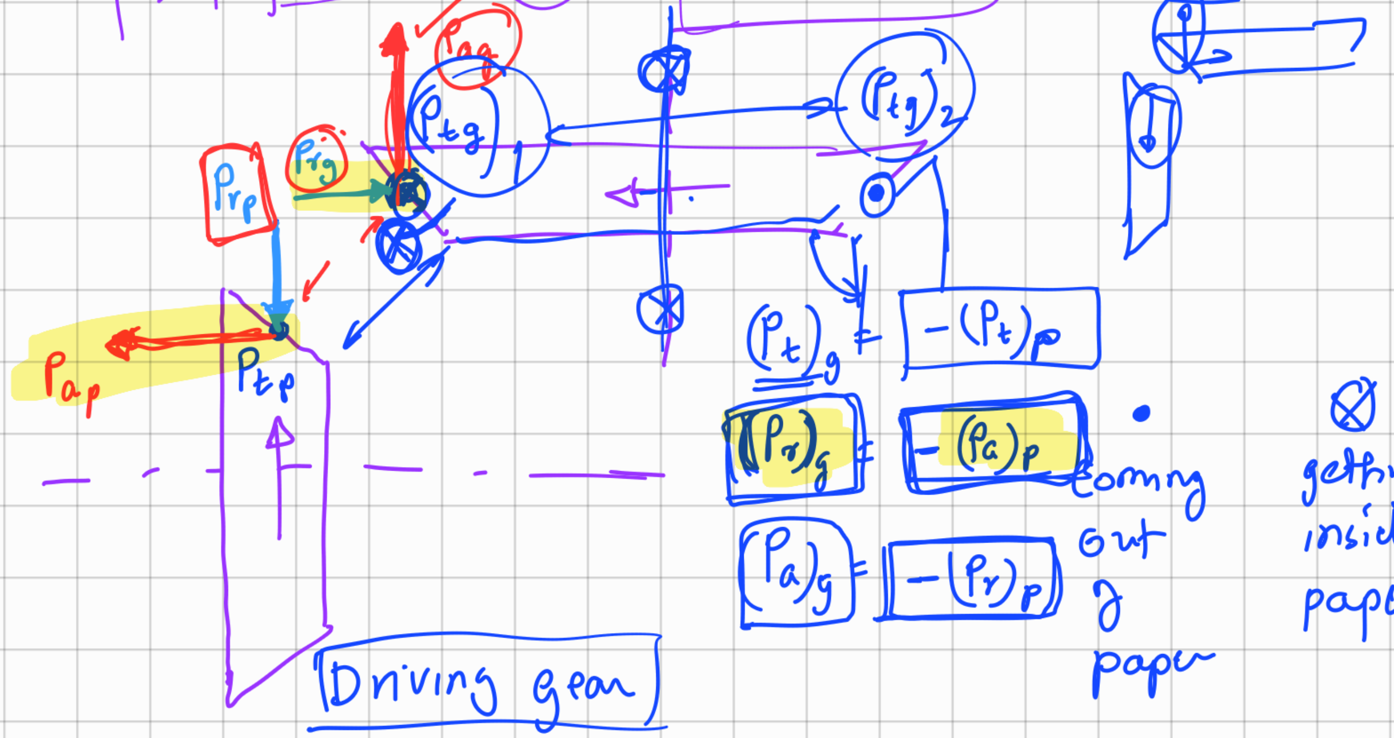
$$\frac{P_a}{P_t} = \frac{\sin\alpha\sin\gamma}{\cos\alpha}$$

$\Rightarrow P_a = P_t \cdot \tan\alpha \cdot \sin\gamma$



$$r_m = \frac{D_p}{2} - \frac{b}{2}\sin\gamma$$

$(P_r)_p \,||\, (P_r)_g$



$(P_t)_g = -(P_t)_p$

$(P_r)_g = -(P_a)_p$

$(P_a)_g = -(P_r)_p$

• coming out of paper

⊗ getting inside paper

QUIZ

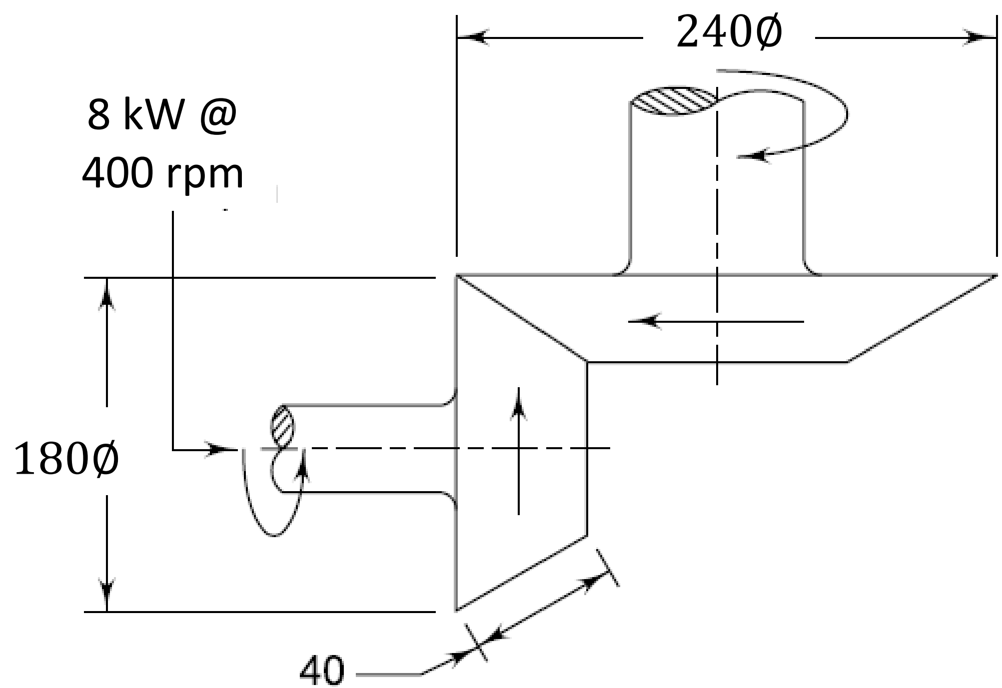
Force Analysis of Bevel Gears

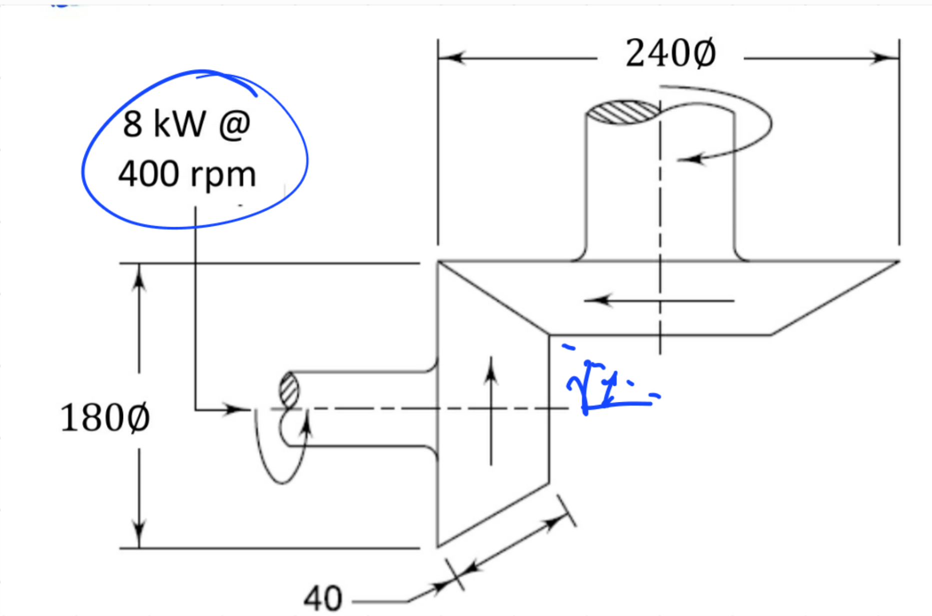
The direction of thrust force on the pinion bevel gear tooth is determined from __________

1. The direction of tangential force for mating wheel
2. The direction of radial force for mating wheel
3. None of the above

Example – Force Analysis

A pair of bevel gears transmitting 8 kW at 400 rpm is shown. The pressure angle is 20 deg. Determine gear tooth forces.





$$M_t = \frac{8 \times 10^6}{\left(\frac{2\pi \times 400}{60}\right)}$$

$$= 190986 \text{ N-mm}$$

$$\tan\gamma = \frac{mZ_p}{mZ_g} = \frac{D_p}{D_g}$$

$$= \frac{180}{240}$$

$$\Rightarrow \boxed{\gamma = 36.9^\circ}$$

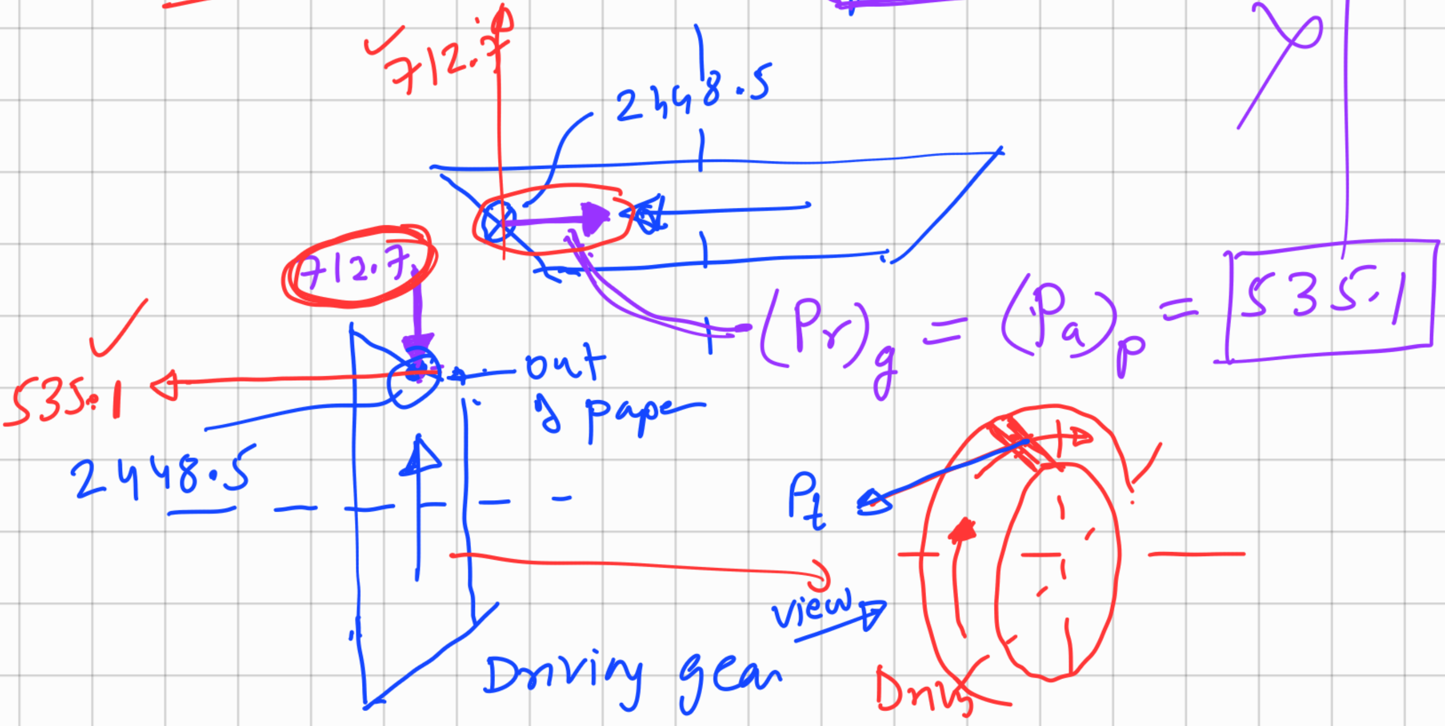
$$r_m = \frac{D_p}{2} - \frac{b}{2} \cdot \sin\gamma$$

$$= \frac{180}{2} - \frac{40}{2} \cdot \sin 36.9^\circ = \boxed{78.0 \text{ mm}}$$

$$P_t = \frac{M_t}{r_m} = \frac{190986}{78} = \boxed{2448.5 \text{ N}}$$

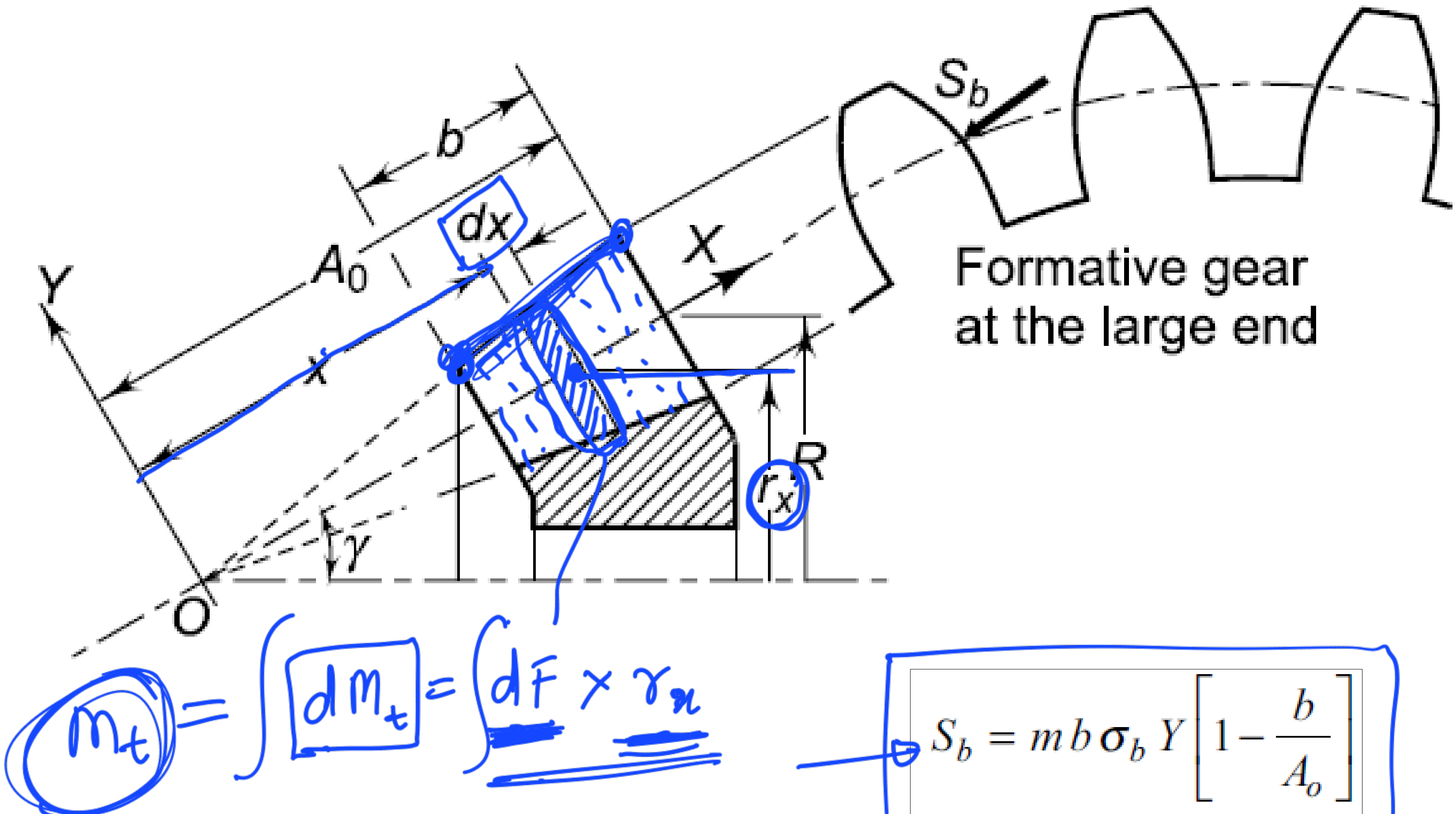
$$P_r = P_t \tan\alpha \cos\gamma = 2448.5 \times \tan 20 \cdot \cos 36.9^\circ = \boxed{712.7 \text{ N}}$$

$$P_a = P_t \tan\alpha \sin\gamma = \boxed{535.1 \text{ N}}$$



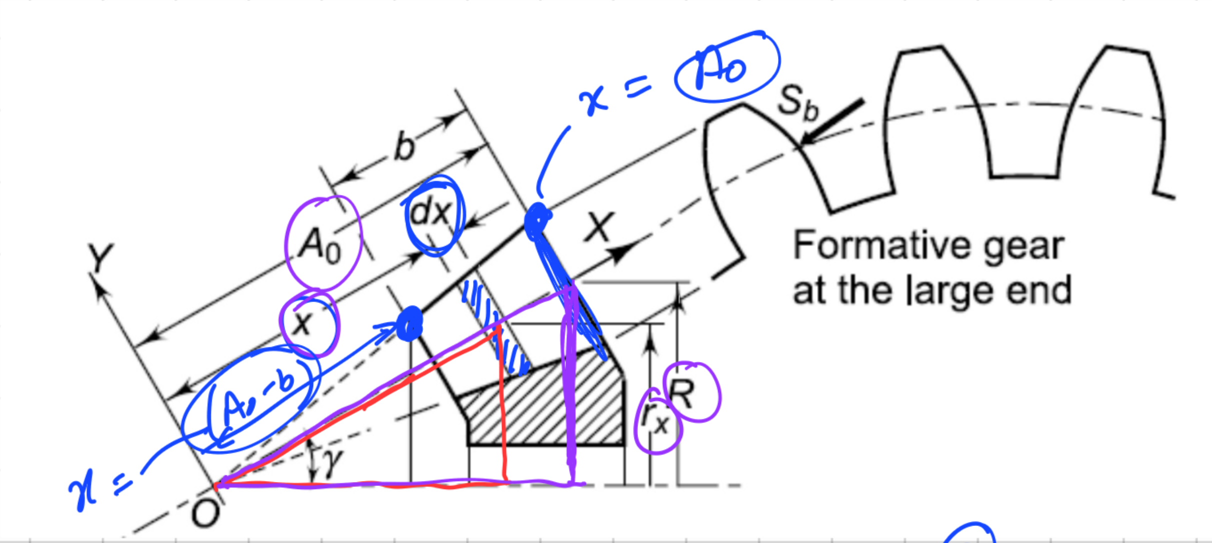
$$(P_r)_g = (P_a)_p = \boxed{535.1}$$

Beam Strength of Bevel Gears (DDB 19.4)



$$S_b = m\,b\,\sigma_b\,Y\left[1 - \frac{b}{A_o}\right]$$

Ref: Bhandari V.B., Design of Machine Elements (2017)



From Lewis formula

$$d(S_b) = \frac{m_x}{P} \cdot b_x \cdot \sigma_b \cdot Y$$

$(D)/z$ ✓

$b_x \rightarrow dx$

$Y \rightarrow$ based on virtual no. of teeth $z' = \frac{z}{\cos\gamma}$

From figure, $\frac{r_x}{R} = \frac{x}{A_0}$

$$\therefore r_x = \frac{xR}{A_0}$$

At the elemental section

$$m_x = \frac{2r_x}{z} = \frac{2xR}{zA_0} \quad \text{--- (1)}$$

At large end of section

$$m = \frac{2R}{z} \quad \text{--- (2)}$$

From (1) & (2)

$$m_x = m\left(\frac{x}{A_0}\right) \quad \text{--- (3)}$$

For elemental slice, $b_x = dx$

$$\therefore d(S_b) = \frac{m\,\sigma_b\,Y\,x\,dx}{A_0}$$

Elemental torque, $d(M_t) = r_x \cdot d(S_b)$

$$\therefore d(M_t) = \frac{x R}{A_0} \times \frac{m \sigma_b Y . x . dx}{A_0}$$

$$M_t = \int_{A_0 - b}^{A_0} \left(\frac{m \sigma_b Y . R}{A_0^2} \right) . x^2 . dx$$

$$= \frac{m \sigma_b Y R}{A_0^2} \left[\frac{x^3}{3} \right]_{(A_0 - b)}^{A_0}$$

$(A_0 - b)^3$

$$= \frac{m \sigma_b Y R}{A_0^2} \left[\frac{A_0^3}{3} - \frac{A_0^3 - 3A_0^2 b + 3A_0 b^2 - b^3}{3} \right]$$

$$\Rightarrow M_t = m b \sigma_b . Y . R \left[1 - \frac{b}{A_0} + \frac{b^2}{3A_0^2} \right]$$

Bending strength of bevel gear is computed at large end of gear

$$\Rightarrow M_t = S_b . R \quad , \quad R = \frac{D_p}{2}$$

$$S_b = m . b . \sigma_b . Y \left[1 - \frac{b}{A_0} + \frac{b^2}{3A_0^2} \right]$$

Ignored

Usually $b < \frac{A_0}{3}$ $\quad < \frac{1}{27}$

@ large end

$$S_b = m b \sigma_b Y \left[1 - \frac{b}{A_0} \right]$$

Bevel factor