F_{a_1}, F_{r_1}, N_1
F_{a_2} F_{r_2} N_2

$P = XVF_r + YF_a$

Design of Machines and Mechanical Systems (PC-BTM711)

Session 09

Module 2: Rolling Contact Bearings – Combined Loadings and Reliability

Session Outcomes

- Bearing design for combination of loads and speeds
- Calculate reliability of bearings
- Discuss practical aspects of bearing design

Design for combinations of loads and speeds

Equivalent load for combination of loads and speeds

(DDB 15.10)

$0 < D < 1/2$ 0.5 D

N_1 P_1 → F_{r1}, F_{a1}

N_2 P_2 → F_{r2}, F_{a2}

0 P_2 0.3

$$\boxed{P_e} = \sqrt[3]{\frac{\sum N_i P_i^3}{\sum N_i}}$$

loads $= P_1, P_2, P_3, \ldots P_x$ (P₁ → F_r, F_a)

Speeds $= n_1, n_2, \ldots n_x$

No. of rev. $= N_1, N_2, \ldots N_x \leftarrow$ cycles.

From load life relationship

$$L = \left(\frac{C}{P}\right)^p \text{ mr}$$

$$= \left(\frac{C}{P}\right)^p \times 10^6 \text{ rev,}$$

($p = 3$ for ball bearing)

Fraction of life consumed in one revolution $= \frac{1}{L}$

$L \rightarrow$ Damage 1

$$= \boxed{\frac{P^3}{C^3} \cdot \frac{1}{10^6}}$$

Life consumed by first load (P_1) $= \left(\frac{P_1^3}{C^3} \times \frac{1}{10^6}\right) N_1$

acts for N_1 cycles

$$= \boxed{\frac{N_1 P_1^3}{10^6 C^3}}$$

Similarly life consumed by second load $= \boxed{\frac{N_2 P_2^3}{10^6 \cdot C^3}}$

Life consumed by all loads $= \sum_{i=1}^{x} \frac{N_i P_i^3}{10^6 C^3}$

Let P_e be equivalent load of all load-speed combinations which acts $N = N_1 + N_2 \ldots + N_x$

$$\boxed{\frac{N \cdot P_e^{3}}{\cancel{10^6} \cdot \cancel{\ell^3}}} = \sum_{i=1}^{2} \frac{N_i \cdot P_i^{3}}{\cancel{10^6}\,\cancel{\ell^3}}$$

$$\boxed{P_e = \sqrt[3]{\frac{\sum N_i P_i^{3}}{\sum N_i}}}$$

Example 1: Combined loads and speeds

A single row deep groove ball bearing is subjected to 30 sec work cycle that consists of following two parts.

	Part I	Part II
Duration (s)	10	20
Radial load (kN)	45	15
Axial load (kN)	12.5	6.25
Speed (rpm)	720	1440

The static and dynamic load capacities of ball bearing are 50 kN and 68 kN respectively. Calculate expected life of bearing in hours.

Given $C_0 = 50$ kN, $C = 68$ kN, $V = 1$ (assume)

(i) Equivalent load calculation $P = XVF_r + YF_a$

	Part I	Part II
$\frac{F_a}{F_r} =$	$\frac{12.5}{45} = 0.28$	$\frac{6.25}{15} = 0.42$
$\frac{F_a}{C_0}$	$\frac{12.5}{50} = 0.25$	$\frac{6.25}{50} = 0.125$
e	0.37	≈ 0.31 ✓ $\Rightarrow \frac{F_a}{F_r} > e$
X	1	0.56
Y	0	1.417 (by interpolation)
$P = XVF_r + YF_a$	$P_1 = 45$ kN	$0.56 \times 15 + 1.417 \times 6.25$ $P_2 = 17.26$ kN
$N_i = \left(\frac{\text{duration}}{\text{in second}}\right) \times \frac{\text{rpm}}{60}$	$10 \times \frac{720}{60} = 120$ rev	$20 \times \frac{1440}{60} = 480$ rev

$N = \sum N_i = N_1 + N_2 = 120 + 480 = 600$ rev (in 30 sec)

Equ. load $= P_e = \sqrt[3]{\frac{\sum N_i P_i^3}{\sum N_i}}$

$= \sqrt[3]{\frac{120 \times 45^3 + 480 \times 17.26^3}{600}}$

$= \boxed{28.16 \text{ kN}}$

(ii) Bearing life Calculation

$$L_{10} = \left(\frac{C}{P_e}\right)^3 = \left(\frac{68}{28.16}\right)^3 = 14.09 \text{ mr}$$

rev.

Life in second $= \dfrac{14.09 \times 10^6}{\left(\frac{600}{30}\right)\cancel{30}} \cdot \dfrac{\text{rev}}{\text{sec}}$

$= 704,500 \text{ sec}$

$= \boxed{195.7 \text{ hr}}$

QUIZ

Combined Loadings

The formula used for combining different load/speed combinations depends on _________

$$P_e = \sqrt[3]{\frac{\sum N_i P_i^3}{\sum N_i}}$$

1. Dynamic capacity of bearing
2. Cumulative damage caused by all loadings
3. Both of above

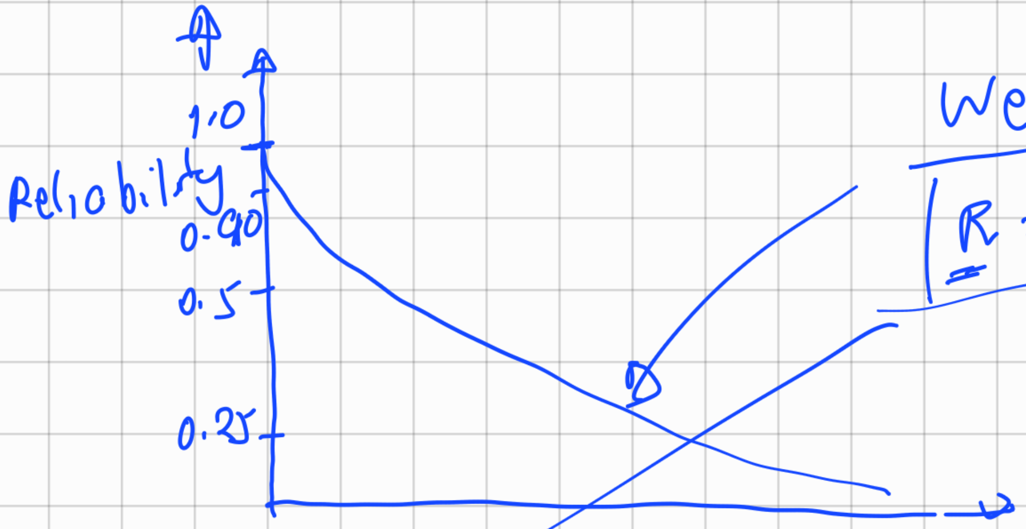
Reliability of bearing

… (DDB 15.16)

$$\frac{L}{L_{10}} = \left[\frac{log_e\left(\frac{1}{R}\right)}{log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

$\underline{R}$ = Reliability = No. of bearings which successfully complete 'L' mr / → Total number of bearings under test

$L_{10} \rightarrow R = 0.90$



Weibull distribution

$$\boxed{R = e^{-\left(\frac{L}{a}\right)^b}}$$

(DDB 15.14)

$$\boxed{L = a \cdot \left[\log_e\left(\frac{1}{R}\right)\right]^{1/b}}$$

$a \rightarrow 6.84$

$b \rightarrow 1.17$

$L_{10} \rightarrow R_{90} = 0.90$

↑ % bearings which will survive

$$\frac{L}{L_{10}} = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

$R_{90} \rightarrow 0.9$

QUIZ

Reliability of Bearings

The expression used for relating life vs reliability of bearing __________

$$\frac{L}{L_{10}} = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

1. is based on Poisson's distribution
2. considers R_{90}=0.1
3. None of above

0.90

Example 3: Reliability of Bearing

A single row deep groove ball bearing is subjected to a radial force of 8 kN and thrust force of 3 kN. X = 0.56 and Y = 1.5. The shaft rotates at 1200 rpm. The shaft diameter is 70 mm and bearing 6314 (C = 104,000 N) is selected for this application.

i. Estimate life of bearing with 90% reliability

ii. Estimate reliability for 10,000 hr life.

$F_r = 8\ kN$

$F_a = 3\ kN$

$$\boxed{\begin{aligned} X &= 0.56 \\ Y &= 1.5 \end{aligned}}$$

$n = 1200\ rpm$

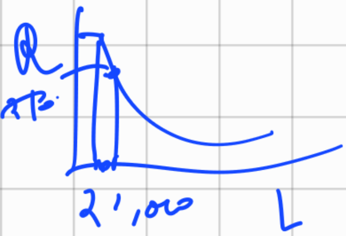
$d = 70\ mm$

bearing $= 6314\ (C = 104{,}000N)$

(i) Life with 90% reliability (L_{10})

$$P = XVF_r + YF_a = 0.56 \times 8 + 1.5 \times 3 = 8.98\ kN$$

$$L_{10} = \left(\frac{C}{P}\right)^p = \left(\frac{104{,}000}{8.98 \times 10^3}\right)^3 = \boxed{1553.36\ mr}$$



$$L_{10h} = \frac{1553.36 \times 10^6}{1200 \times 60} = \boxed{21{,}574\ hr}$$

(ii) Reliability for 10,000 hr life

$$\frac{L}{L_{10}} = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

$$\frac{10{,}000}{21{,}574} = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{0.9}\right)}\right]^{1/1.17}$$

$$\boxed{R = 0.9581}$$

SELF-STUDY

Practical aspects of bearing design

- Bearing failure – Causes and remedies
- Lubrication of rolling contact bearings
- Mounting arrangement for bearings