Design of Machines and Mechanical Systems (PC-BTM711)

Session 06

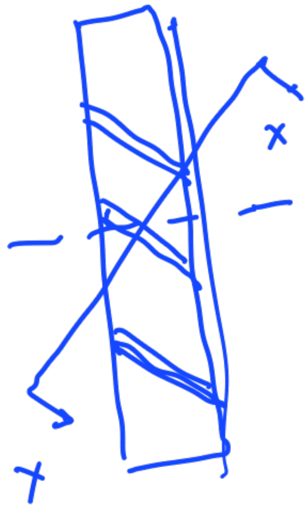
Module 1: Helical Gear Design

Session Outcomes

- Derive design equations for helical gears
- Perform design calculations for helical gears

QUIZ

Formative/Virtual Gear



Formative gear for helical gear is formed by ___________

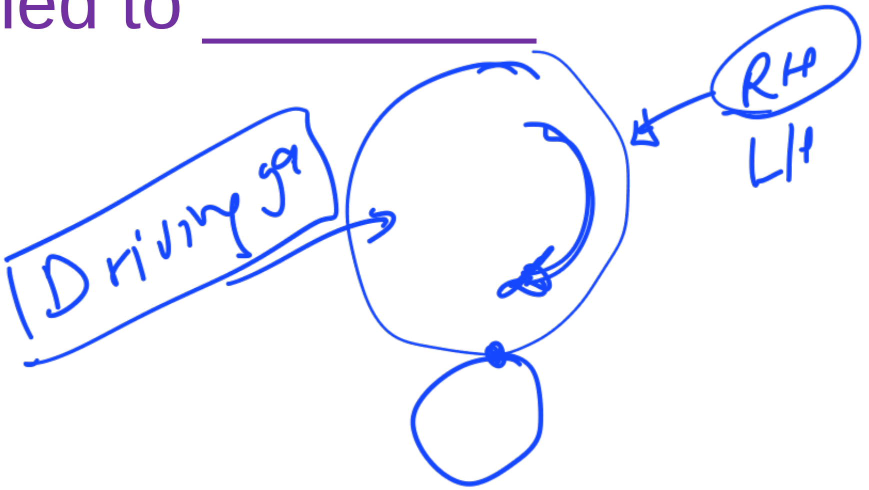
1. Taking a cross-section normal to the direction of helix
2. Taking a cross-section along the direction of helix

QUIZ

Axial Force in Helical Gear

For establishing the direction of axial force, the RH/LH thumb rule is applied to __________

1. Driven Gear
2. Driving Gear



Helical Gear – Design Equations

Table 18.6 *Beam strength of gear tooth (Lewis' equation)*

Beam strength (S_b) indicates the maximum value of tangential force that the tooth can transmit without bending failure.		
$S_b = m_n b \sigma_b Y$	(18.17)	S_b = beam strength of gear tooth (N) σ_b = permissible bending stress (MPa or N/mm^2) Y = Lewis form factor based on virtual number of teeth (z') (Table 17.15)
$\sigma_b = S_e = \left(\frac{1}{3}\right) S_{ut}$	(18.18)	S_e = endurance limit (MPa or N/mm^2) S_{ut} = ultimate tensile strength (MPa or N/mm^2)

Table 18.7 *Wear strength of gear tooth (Buckingham's equation)*

Wear strength (S_w) indicates the maximum value of tangential force that the tooth can transmit without pitting failure.		
$S_w = \frac{bQd_pK}{\cos^2 \psi}$	(18.19)	S_w = wear strength of the gear tooth (N) Q = ratio factor d_p = pitch circle diameter of pinion (mm) K = load – stress factor (MPa or N/mm^2) ψ = helix angle (°)

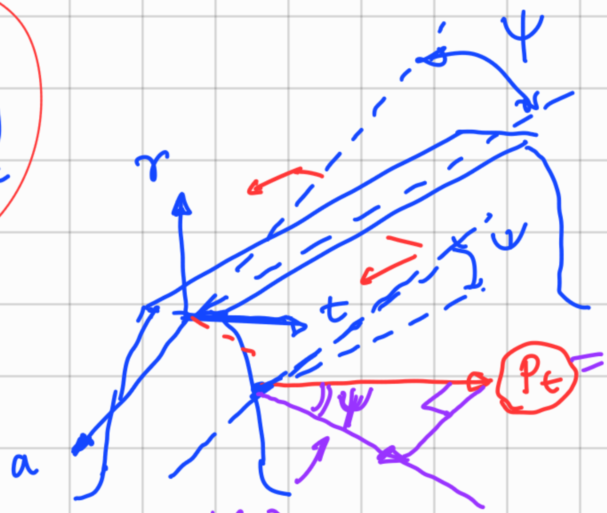
$(P_t)_{max}$

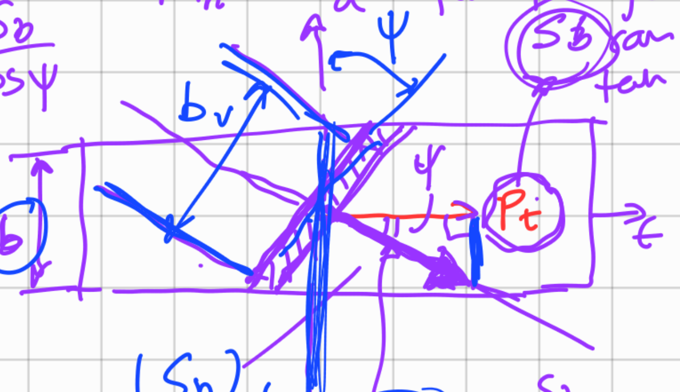
$$(S_b)_v = (m)_v (b)_v \sigma_b (Y)_v$$

m_n

$\frac{S_b}{\cos\psi}$

max. tang. force helical gear

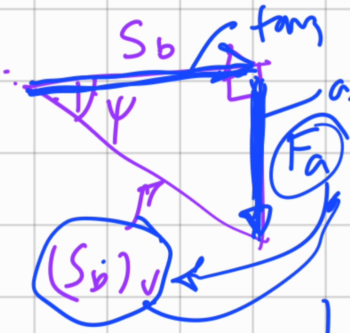




$$b_v = \frac{b}{\cos\psi}$$

$(S_b)_v$

$(S_b)_v = \frac{S_b}{\cos\psi}$



max. Tang. force that virtual teeth can take.

$$\frac{S_b}{\cos\psi} = m_n \cdot \frac{b}{\cos\psi} \cdot \sigma_b Y$$

Y for virtual number of teeth

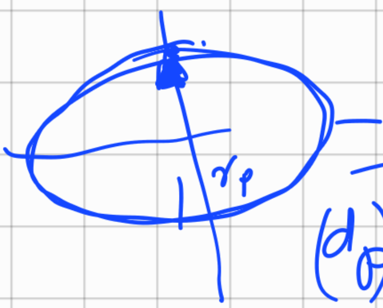
$$z' = \frac{z}{\cos^3\psi}$$

$$S_b = m_n \cdot b \cdot \sigma_b \cdot Y$$

Wear strength of helical gear

max. tang. force on virtual gear → $(S_w)_v = (b)_v (Q)_v (d_p)_v K$

$$\frac{S_w}{\cos\psi} = \frac{b}{\cos\psi} \times Q \times \frac{d_p}{\cos^2\psi} \times K \quad \frac{z_g}{\cos^3\psi}$$



$$(d_p)_v = \frac{d_p}{\cos^2\psi}$$

$$(Q)_v = \frac{2z_g'}{z_p' + z_g'} = \frac{2z_g}{z_p + z_g}$$

$\frac{z_p}{\cos^3\psi}$

$$z' = \frac{z}{\cos^3\psi}$$

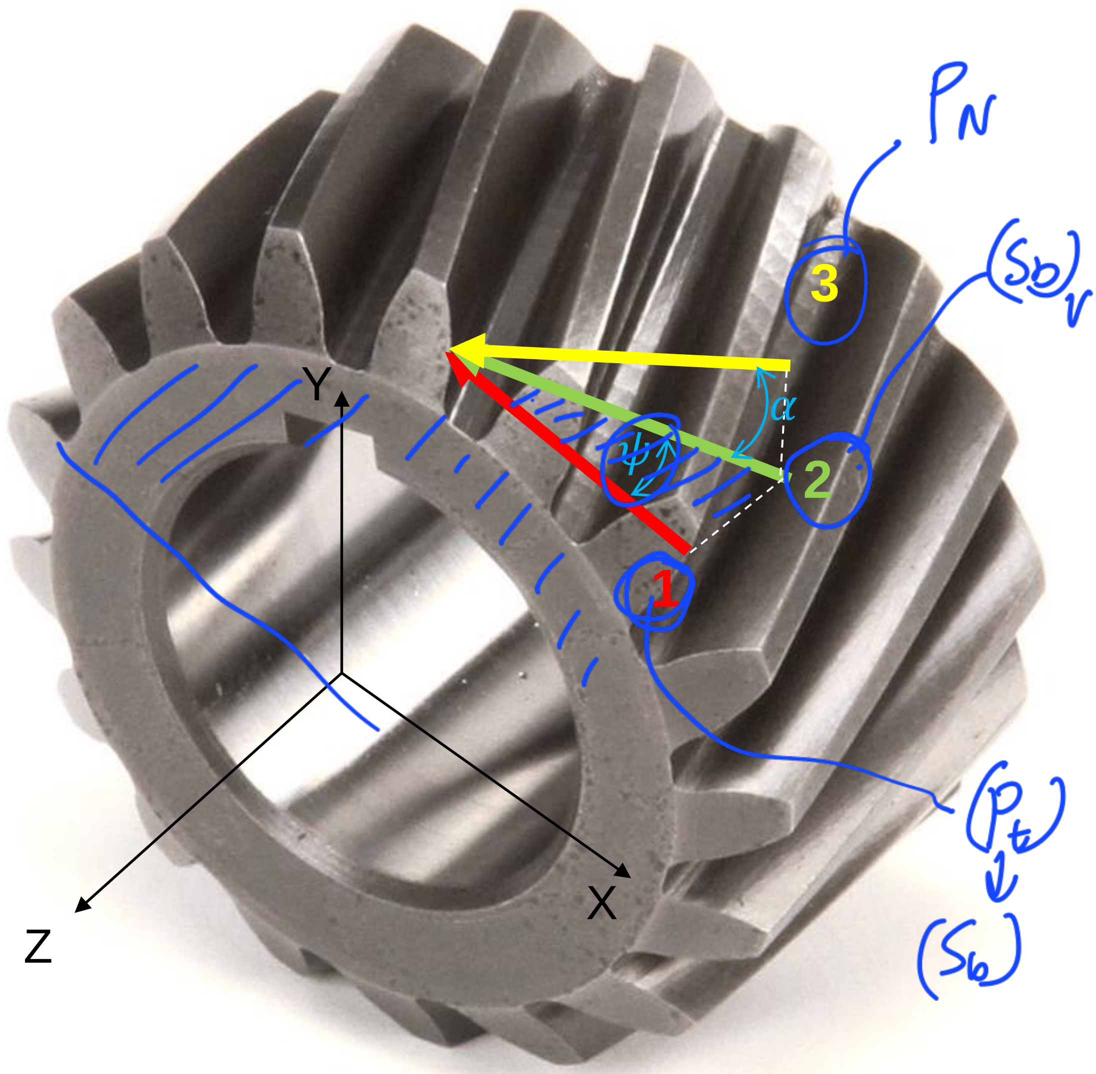
$$S_w = \frac{b \cdot Q \cdot d_p \cdot K}{\cos^2 \psi}$$

QUIZ

Direction of S_b and S_w

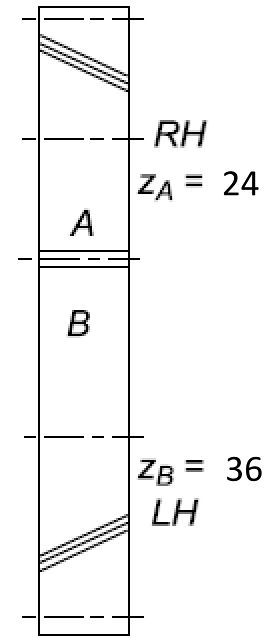
The direction of S_b or S_w considered in strength equations is ________

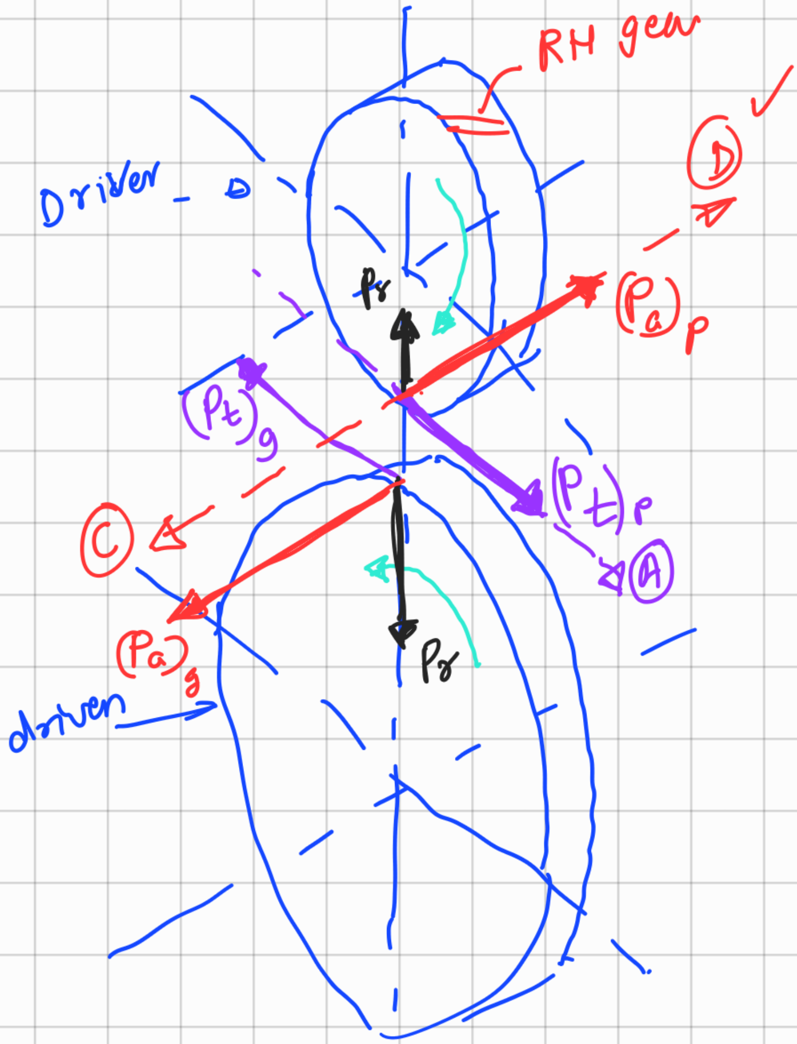
1. Vector 1 lying along X-axis
2. Vector 2 lying in plane XZ
3. Vector 3 lying in plane formed by Y-axis and Vector 2.



Ex.1: Force Analysis for helical gears

A pair of parallel helical gears are shown in figure. A 6 kW power at 800 rpm is supplied to pinion 'A' through its shaft. Normal module is 5 mm and normal pressure angle is 20 deg. The pinion has right handed teeth while gear is left handed. The helix angle is 30 deg. The pinion rotates in clockwise direction when seen from left side of figure. Determine the tooth force components.





$KW = 6$

$n = 800$

$m_n = 5\ mm$

$\alpha_n = 20^\circ$

$\psi = 30^\circ$

$z_p = 24$

$z_g = 36$

$$M_t = \frac{60 \times 6 \times 10^6}{2\pi \times 800} = 71619.7 \text{ N.mm}$$

$$d_p = \frac{z_p \cdot m_n}{\cos\psi} = \frac{5 \times 24}{\cos 30^\circ} = 138.6 \text{ mm}$$

$$P_t = \frac{2 \times 71619.7}{138.6} = 1033.5\ N$$

$$P_r = \frac{P_t \cdot \tan\alpha_n}{\cos\psi} = \frac{1033.5 \times \tan 20^\circ}{\cos 30^\circ}$$

$$= 434.4\ N$$

$$P_a = P_t \cdot \tan\psi = 596.7\ N$$

Ex.2: Strength Design for helical gears

A pair of parallel helical gears consists of a 24 teeth pinion meshing with 120 teeth gear. The pinion rotates at 600 rpm. The normal pressure angle is 20 deg., helix angle is 30 deg. The face width is 50 mm and the normal module is 5 mm.

The pinion as well as gear is made of forged steel with UTS = 600 MPa and heat treated to surface hardness of 350 BHN. The service factor and factor of safety are 1.25 and 2.5 respectively.

Assume that velocity factor accounts for the dynamic load and calculate the power transmitting capacity of the gears.

$Z_p = 24$
$Z_g = 120$
$n_p = 600$ rpm
$\alpha_n = 20°$
$\psi = 30°$
$b = 50$ mm
$m_n = 5$ mm
UTS = 600 MPa
BHN = 350
$C_s = 1.25$
FOS = 2.5
Vel. factor method
kW = ?

S_b S_w P_{eff} kW ?

(1) Calculation of bending strength

$$S_b = m_n \cdot b \cdot \sigma_b \cdot Y_a$$ ✓

$$z' = \frac{z}{\cos^3\psi} = \frac{24}{\cos^3 30°} = 36.95$$

$z = 35$ $Y = 0.373$
$z = 37$ $Y = 0.380$ } DDB T17.15

$\Rightarrow$ $Y = 0.3798$ (By interpolation)

$$\sigma_b = \frac{UTS}{3} = 200 \text{ MPa}$$

$$S_b = 5 \times 50 \times 200 \times 0.3798$$
$$= 18{,}990 \text{ N}$$

(2) Wear strength Calculation

$$S_w = \frac{b\, Q\, d_p \cdot K}{\cos^2\psi}$$

$$Q = \frac{2 Z_g}{Z_g + Z_p} = \frac{2 \times 120}{24 + 120} = 1.667$$

$$d_p = \frac{Z_p \cdot m_n}{\cos\psi} = \frac{24 \times 5}{\cos 30°} = 138.6 \text{ mm}$$

$$K = 0.16\left(\frac{BHN}{100}\right)^2 = 0.16 \times \left(\frac{350}{100}\right)^2 = 1.96 \text{ N/mm}^2$$

$$S_w = \frac{50 \times 1.667 \times 138.6 \times 1.96}{\cos^2 30°}$$
$$= 30{,}190.0 \text{ N}$$

Since $S_b < S_w \Rightarrow S_b$ is criterion for failure

(c)

$$P_{eff} = \frac{C_s}{C_v} \cdot P_t$$

$$FOS = \frac{\min(S_b, S_w)}{P_{eff}}$$

$$\therefore \quad 2.5 = \frac{18{,}990}{\frac{C_s}{C_v} \cdot P_t}$$

kW = $S_b \times \frac{d_p}{2} \times \omega$

C_s, FOS, $\frac{1}{C_v}$

$$v = \frac{2\pi n_p}{60} \times \frac{d_p}{2} = \frac{2\pi \times 600}{60} \times \frac{138.6 \times 10^{-3}}{2}$$

$$= 4.35 \text{ m/s}$$

$$\Rightarrow C_v = \frac{5.6}{5.6 + \sqrt{v}} \qquad \text{DDB T18.8}$$

$$= \frac{5.6}{5.6 + \sqrt{4.35}} = 0.7286$$

$$2.5 = \frac{18{,}990}{\frac{1.25}{0.7286} \times P_t}$$

$$\Rightarrow \boxed{P_t = 4427.6 \text{ N}}$$

(d) Power calc

$$M_t = \frac{P_t \cdot d_p}{2} = \frac{4427.6 \times 138.6}{2}$$

$$= 306{,}832.7 \text{ N·mm}$$

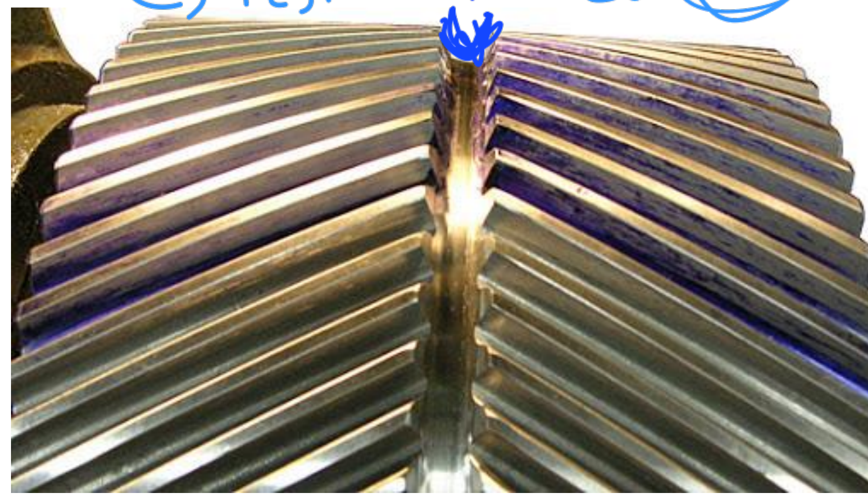
$$kW = M_t \times \frac{2\pi n_p}{60} \times 10^{-6}$$

$$= 306{,}832.7 \times \frac{2\pi \times 600}{60} \times 10^{-6}$$

$$= \boxed{19.28}$$

SELF-STUDY

Herringbone and Double-Helical Gears



Herringbone

Double-Helical gears