S_w P_t  $S_b = m b Y \sigma_b$

Service $C_s P_t$ Dynamic effect

Design of Machines and Mechanical Systems (PC-BTM711)

$P_{eff} < \min(S_b, S_w)$

P_{eff}

Session 04

Module 1: Spur Gear – Design Calculations

Session Outcomes

- Calculate effective load on gear tooth including service factor and dynamic effects
- Describe wear strength-based design
- Perform preliminary spur gear strength design calculations

QUIZ

Dynamic force on gear tooth

Dynamic force on gear tooth arises due to ___

1. Inaccuracies in tooth profile ✓
2. Elasticity of parts ✓
3. Both of above

$P_{eff} < \boxed{S_b}, S_w$

Effective load on gear tooth

- Effective load on gear tooth (Velocity factor method)

Preliminary design

$$\boxed{P_{eff} = \frac{C_s P_t}{C_v}}$$

Limited range

$f(v)$

@PCD

ALTERNATIVELY

- Effective load as per Buckingham equation

low velocity
low mass

Detailed / final design

$$\boxed{P_{eff} = C_s P_t + \boxed{P_d}}$$

Buckingham load.

Buckingham dynamic load

$$P_d = \frac{21v(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}} \qquad (17.32)$$

P_d = dynamic load or incremental dynamic load (N)

v = pitch line velocity (m/s)

C = deformation factor (MPa or N/mm^2) (Tables 17.24 and 17.25)

e = sum of errors between two meshing teeth (mm) (Table17.26)

b = face width of tooth (mm)

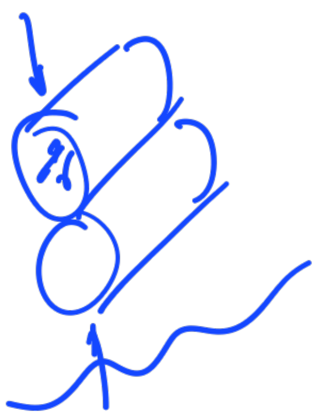
P_t = tangential force due to rated torque (N)

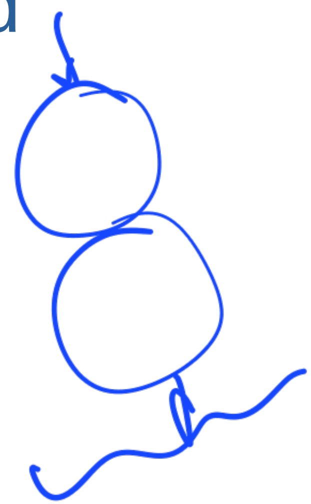
QUIZ

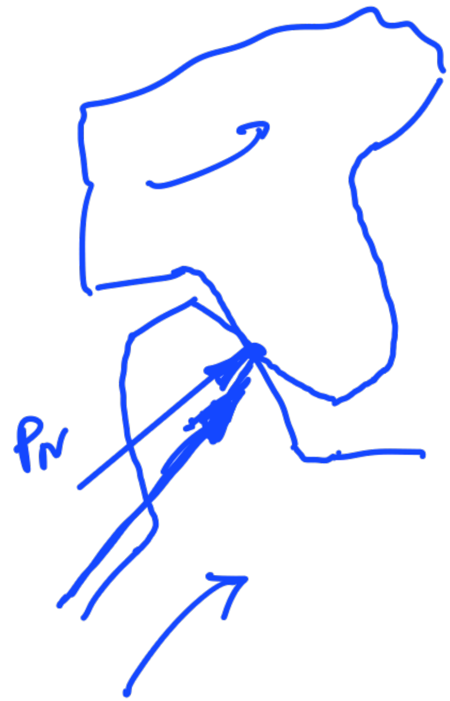
Velocity Factor and Buckingham Load

For preliminary gear design, ________ method is usually used to obtain effective load.

1. Velocity factor
2. Buckingham load

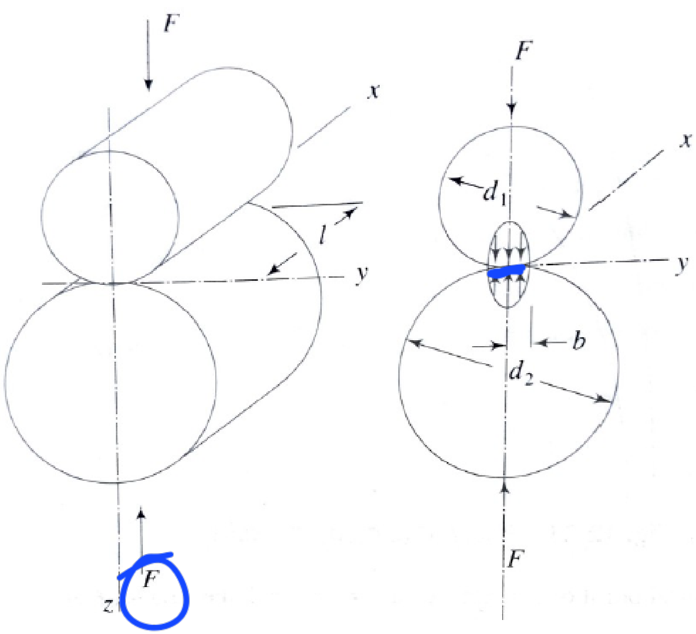






Wear strength of gear tooth

- Basis for derivation of Wear strength equation



$$b = \sqrt{\frac{2F}{\pi l}\left[\frac{\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}}{\frac{1}{d_1}+\frac{1}{d_2}}\right]}$$

$$\boxed{p_{max} = \frac{2F}{\pi b l}}$$

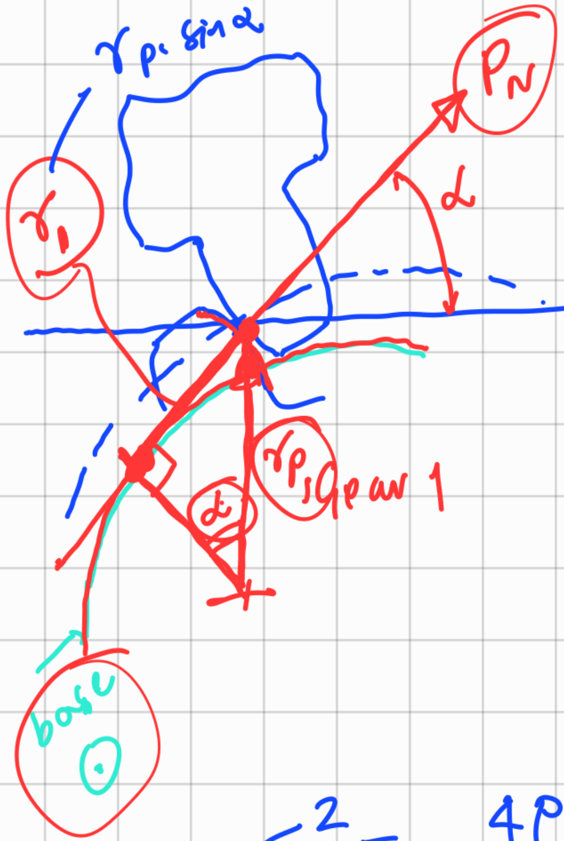
$$\sigma_x = -2\nu p_{max}\left[\sqrt{\left(1+\frac{z^2}{b^2}\right)}-\frac{z}{b}\right]$$

$$\sigma_y = -p_{max}\left[\left(2-\frac{1}{1+z^2/b^2}\right)\sqrt{1+z^2/b^2}-2\frac{z}{b}\right]$$

$$\sigma_z = -p_{max}\left[\frac{1}{\sqrt{1+z^2/b^2}}\right]$$

9

Credits: Srinath L.S., Advanced Mechanics of Solids (2009)



$$\sigma_c = \frac{2P}{\pi b l}$$

poisson's

$$b = \left[\frac{2P(1-\mu^2)\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}{\pi l\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}\right]^{1/2}$$

$$\sigma_c^2 = \frac{4P^2}{\pi^2 b^2 l^2}$$

$$\sigma_c^2 = \frac{4P^2}{\pi^2 l^2} \times \frac{\pi l\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}{2P(1-\mu^2)\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}$$

P_N — P_t — S_w

$$\sigma_c^2 = \frac{1}{\pi(1-\mu^2)}\left(\frac{P}{l}\right)\left[\frac{\frac{1}{r_1}+\frac{1}{r_2}}{\frac{1}{E_1}+\frac{1}{E_2}}\right]$$

We have $r_1 = \frac{d_p}{2} \cdot \sin\alpha$

$r_2 = \frac{d_g}{2} \cdot \sin\alpha$

$$\frac{1}{r_1}+\frac{1}{r_2} = \frac{2}{\sin\alpha}\left(\frac{1}{d_p}+\frac{1}{d_g}\right)$$

$$Q = \text{Ratio factor} = \frac{2 Z_g}{Z_p + Z_g} = \frac{2 d_g}{d_p + d_g}$$

$$\Rightarrow \frac{1}{d_p}+\frac{1}{d_g} = \frac{2}{Q \cdot d_p}$$

$$\boxed{\frac{1}{r_1} + \frac{1}{r_2} = \frac{4}{Q\, d_p \cdot \sin\alpha}}$$

gear tooth width.

$$p = P_N = \frac{P_t}{\cos\alpha}, \quad l = b, \quad \mu = 0.3$$

$$\sigma_c^2 = \frac{1.4\, P_t}{b Q\, d_p \sin\alpha \cos\alpha \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}$$

$$P_t = b Q d_p \times \underbrace{\frac{\sigma_c^2 \cdot \sin\alpha\cos\alpha\left(\frac{1}{E_1} + \frac{1}{E_2}\right)}{1.4}}_{\text{"K"}}$$

"K" ↳ stress factor

$$\boxed{S_W = b Q d_p \cdot K}$$

$$K = 0.16\left(\frac{BHN}{100}\right)^2$$

$$\boxed{S_b = m b Y \sigma_b}$$

P_{eff}: $\dfrac{C_s P_t}{C_v}$ or $C_s \cdot P_t + P_d$

↳ smaller than $\min(S_w, S_b)$

m

KW, rpm, α

Gear design → Check design; design from scratch

QUIZ

Wear Strength - Ratio Factor Q

In wear strength calculations, ratio factor Q is ___.

1. $\frac{2z_g}{z_g+z_p}$ ✓ → external gear

2. $\frac{2z_g}{z_g-z_p}$ ← internal gears

QUIZ

Wear Strength – Load factor K

In wear strength calculations, load factor K is related to _____.

1. Surface endurance strength
2. Surface hardness
3. Both of above

$K = 0.16\left(\frac{BHN}{100}\right)^2$

Estimation of module based on bending strength

Table 21.25 *Formulae based on beam strength of pinion tooth (Bending strength)*

Formulae for designing gears		
Spur gears		
$m \geq 1.26 \sqrt[3]{\dfrac{[M_t]}{y[\sigma_b]\psi_m Z_1}}$	(21.11)	m = module of gear tooth (mm) $[M_t]$ = design twisting moment or design torque (N-mm) (Table 21.9) y = Lewis form factor (Table 21.26) $[\sigma_b]$ = design bending stress (MPa or N/mm^2) (Table 21.15) $\psi_m = \left(\dfrac{b}{m}\right)$ factor [ψ_m is usually 8 to 12] [$b \cong 10\ m$] b = face width of gear (mm) Z_1 = number of teeth on pinion

Example 1: Design based on velocity factor method

It is required to design a pair of spur gears with 20^{o} full-depth involute teeth based on Lewis equation.

The velocity factor is to be used to account for dynamic load.

The pinion shaft is connected to a 10 kW, 1440 rpm motor. The starting torque of motor is 150% of rated torque. The speed reduction is 4:1.

C_s $\frac{150}{100}$

Nm

1.5

1 Nm

The pinion as well as gear is made of plain carbon steel 40C8 (UTS = 600 MPa). The factor of safety can be taken as 1.5.

Design the gears, specify their dimensions and suggest suitable surface hardness for the gears.

$kW = 10$ $n = 1440$ rpm, $\boxed{i = 4}$, UTS = 600 MPa

$C_s = 1.5$ $\boxed{FOS = 1.5}$ the factor of

equal to starting torque of motor

For $\alpha = 20^\circ$, $z_{min} = 17$ (DDB T 17.12)

Let $z_p = 17$

$z_g = 4 \times 17 = 68$

$$\rightarrow M_t = \frac{60 \times 10^6 \, kW}{2\pi n_p} = \frac{60 \times 10^6 \times 10}{2\pi \times 1440} = 66,314.6 \text{ N·mm}$$

~~For~~ $z = 17$, $Y = 0.302$ (DDB T 17.15)

Not T21.26

$$m \geq 1.26 \sqrt[3]{\frac{[M_t]}{Y [\sigma_b] \psi_m z_1}}$$

$$[M_t] = \frac{C_s \cdot M_t}{C_v} \times \boxed{FOS}$$

$C_s = 1.5$

$\boxed{\frac{b}{m} = 10}$ From DDB 17.22, $C_v = \frac{3}{3 + v_p}$

between 8 - 12

$\frac{m z_p}{2} \leftarrow \frac{d_p}{2} \rightarrow r_p \cdot \omega_p$

Assumed (arbitrarily) $v = 5$ m/s

$$\Rightarrow C_v = \frac{3}{3 + 5} = 0.375$$

$$[M_t] = \frac{C_s \cdot M_t}{C_v} \times FOS$$

$$= \frac{1.5 \times 66{,}314.6}{0.375} \times 1.5$$

$$= 397{,}887.6 \text{ N.mm}$$

$$[\sigma_b] = \frac{UTS}{3} = \frac{600}{3} = 200 \text{ N/mm}^2$$

$$\psi_m = b/m = 10$$

$$z_1 = 17$$

$$m \geq 1.26 \sqrt[3]{\frac{397{,}887.6}{0.302 \times 200 \times 10 \times 17}}$$

$$\geq 4.26 \text{ mm}$$

Selection of module

From DDB T 17.3, $m = 5$ (choice 1)

$$d_p = m z_p = 5 \times 17 = 85 \text{ mm}$$

$$d_g = m z_g = 5 \times 68 = 340 \text{ mm}$$

$$b = 10\,m = 50 \text{ mm}$$

$\boxed{\text{Effective load Calculation}}$

$$P_t = \frac{2M_t}{d_p} = \frac{2 \times 66{,}314.6}{85} = 1560.3 \text{ N}$$

$$v = \frac{\pi d_p n_p}{60 \times 10^3} = \frac{\pi \times 85 \times 1440}{60 \times 10^3} = \underline{\underline{6.41 \text{ m/s}}}$$

$$C_v = \frac{3}{3+v} = \frac{3}{3+6.41} = 0.3188$$

$$P_{eff} = \frac{C_s \cdot P_t}{C_v} = \frac{1.5 \times 1560.3}{0.3188} = \boxed{7,341.4\ N}$$

Bending strength

$$\boxed{S_b = m\ b\ \sigma_b\ Y}$$

$Y \propto z$

$Y_g > Y_p$

$\Rightarrow (S_b)_g > (S_b)_p$ ✓

$$= 5 \times 50 \times 200 \times 0.302$$

$$= \boxed{15,100\ N}$$

$$\text{Actual FOS} = \frac{S_b}{P_{eff}} = \frac{15,100}{7341.4} = 2.06$$

Since actual FOS (2.06) > 1.5 ⇒ okay