

Design of Machines and Mechanical Systems (PC-BTM711)

Session 03

Module 1: Spur Gear – Bending Strength

Session Outcomes

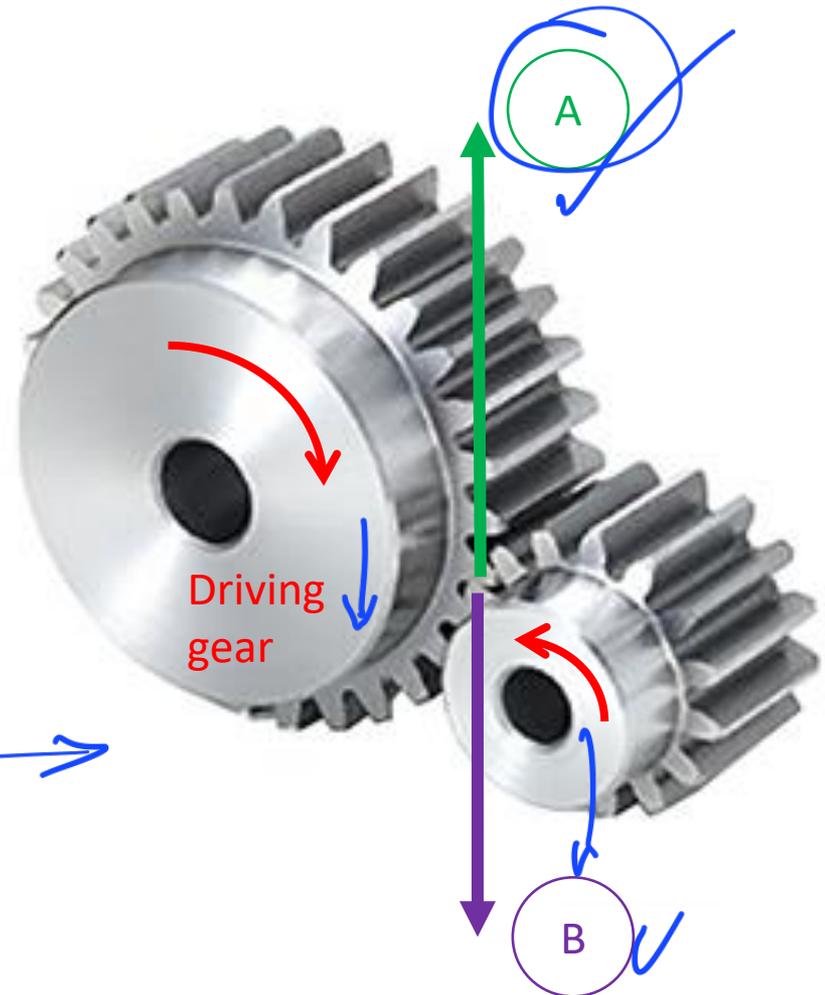
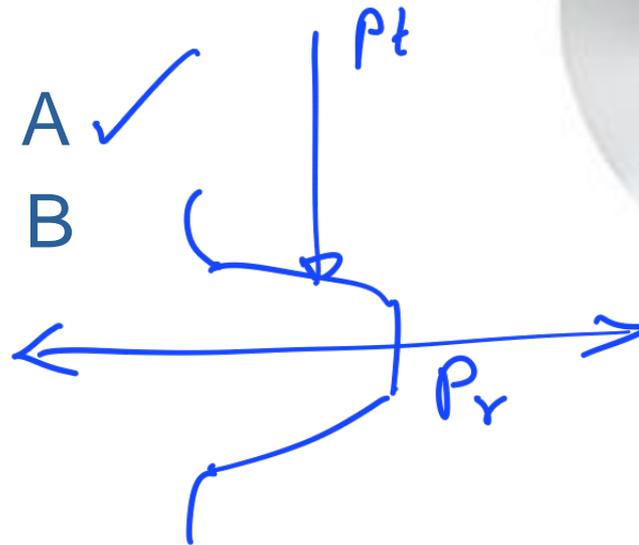
- Calculate forces on spur gear tooth
- Discuss modes of gear tooth failure
- Describe selection of material for gears
- Calculate Beam strength of gear tooth
- Calculate effective load on gear tooth including service factor and dynamic effects

QUIZ

Force Analysis – Spur Gears

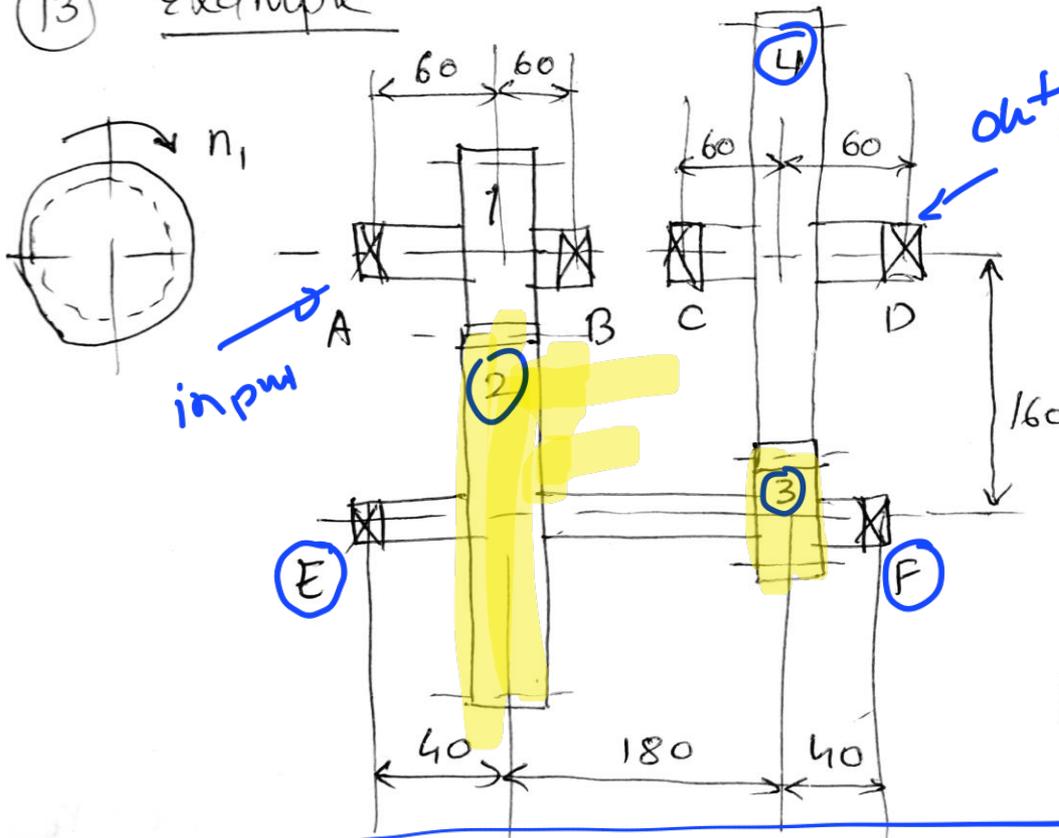
The direction of force acting on the tooth of driving gear is _____

1. Direction A ✓
2. Direction B



Example 2: Spur gear force analysis

⑬ Example



output

$\alpha = 20^\circ$

$z_1 = 20$

$z_2 = 60$

$z_3 = 20$

$z_4 = 60$

$n_1 = 800 \text{ rpm}$

$KW = 20$

Determine reactions on bearings

Determine reactions on bearings E and F.

(a) Module of gear

$$a = \frac{m(z_p + z_g)}{2}$$

$$m = \frac{2a}{z_p + z_g}$$

$$= \frac{2 \times 160}{20 + 60}$$

$$= \boxed{4 \text{ mm}}$$

○

(b) $d_1 = 4 \times 20 = \boxed{80 \text{ mm}}$
 $d_2 = 4 \times 60 = 240 \text{ mm}$

(c) Forces on gears 1-2

$(M_t)_1 = \frac{60 \times 20 \times 10^6}{2\pi \times 800}$
 $= 238.7 \times 10^3 \text{ N.m}$

$(P_t)_1 = (P_t)_2$
 $= \frac{2 \times 238.7 \times 10^3}{80}$
 $= \boxed{5967.5 \text{ N}}$

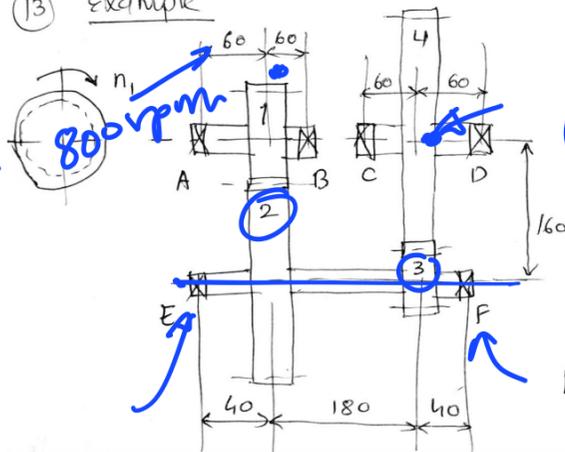
$(P_r)_1 = (P_r)_2 = (P_t)_1 \times \tan \alpha = \boxed{2172 \text{ N}}$

(d) Forces in gear pair 3-4

$(M_t)_3 = \frac{60 \times 10^6 \text{ kW}}{2\pi \times 1800}$

$n_2 = n_3 = \frac{20}{60} \times 800$
 $= 266.7 \text{ rpm}$

(13) Example



$\alpha = 20^\circ$
 $z_1 = 20$
 $z_2 = 60$
 $z_3 = 20$
 $z_4 = 60$
 $n_1 = 800 \text{ rpm}$
 $KW = 20$
Determine reactions on bearings

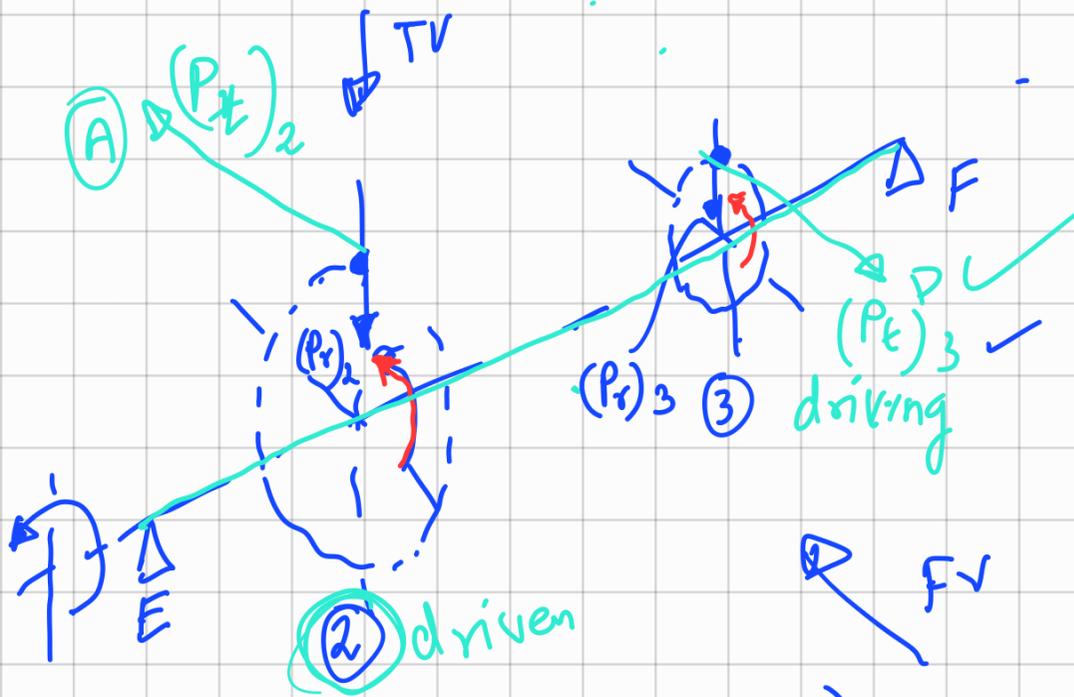
Determine reactions on bearings E and F.

$$(m_t)_3 = \frac{60 \times 10^6 \times 20}{2\pi \times 266.7} = 716.1 \times 10^3 \text{ N}\cdot\text{mm}$$

$$(P_t)_3 = \frac{2 \times (716.1 \times 10^3)}{80} \quad d_3 = m \times z_3$$

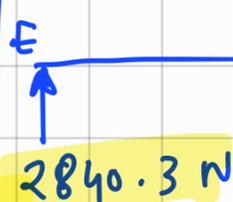
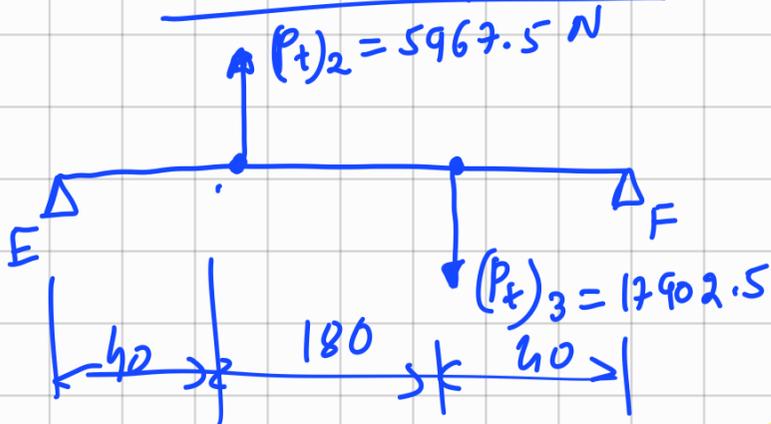
$$= 17902.5 \text{ N}$$

$$\checkmark (P_r)_{t3} = (P_t)_3 \tan \alpha = 6516 \text{ N}$$



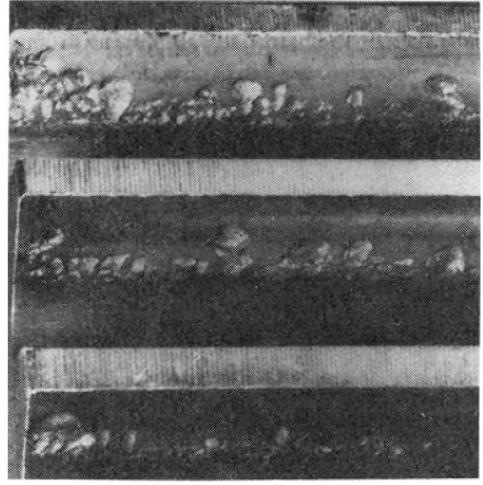
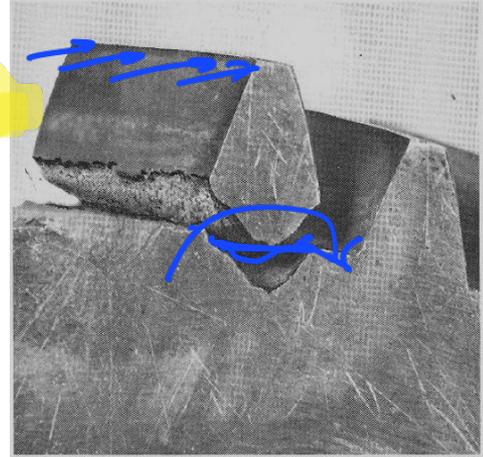
Horizontal plane (TV)

Vertical plane (FV)



Modes of failure of gear tooth

- Breakage of tooth or bending failure
- Surface destruction or Wear failure
 - Abrasive wear (due to foreign particles)
 - Corrosive wear (due to chemical action)
 - Initial pitting (initial corrections at high spots)
 - Destructive pitting (surface fatigue failure)
 - Scoring (alternate welding and shearing at high spots)



Selection of material for gears

- UTS/YS, endurance strength
- Wear resistance, surface fatigue strength
- Coefficient of friction
- Extent of thermal distortion during heat treatment
- Metallic and non-metallic materials

Bending strength of gear tooth

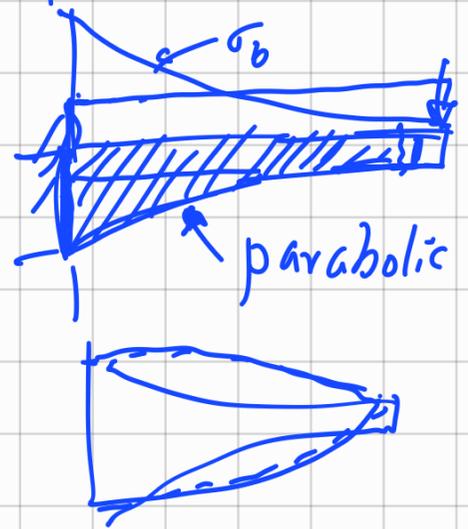
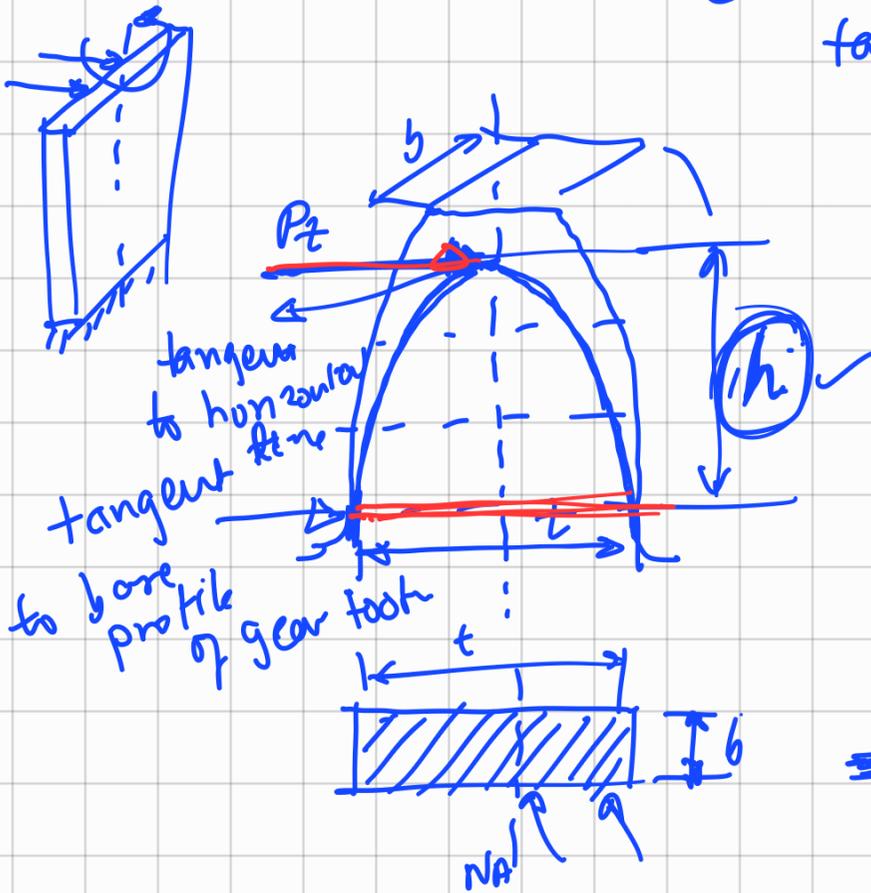
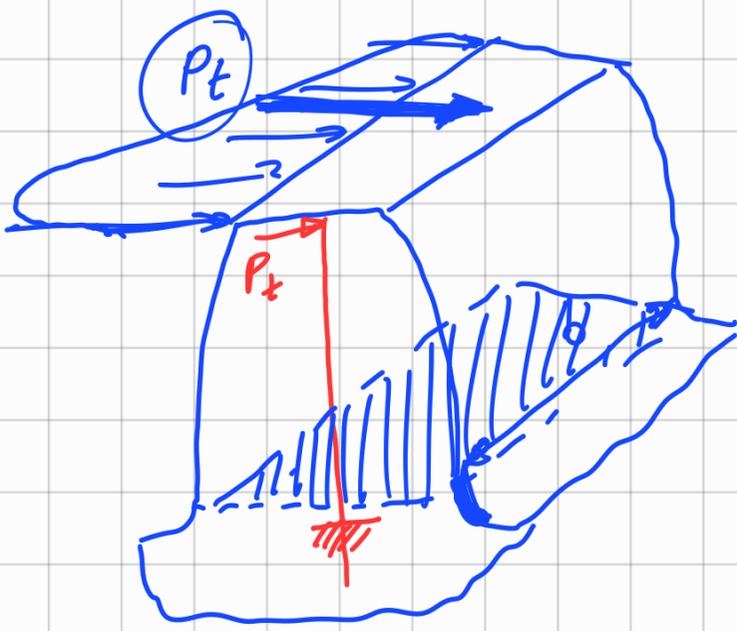
- Derivation of Lewis equation

- Allowable bending stress

(Lewis eqn
(year 1892))

Assumptions

- ① Radial force is neglected
- ② P_t is archy uniformly over width of tooth
- ③ stress conc. effect is neglected
- ④ Only one point of tooth takes full load at a time



$$\sigma_b = \frac{m \odot}{I} \text{ allowable tens. force}$$

$$= \frac{(P_t \times h) \times t/2}{\frac{1}{12} b t^3}$$

Allowable tang. force $\rightarrow P_t = b \cdot \sigma_b \left(\frac{t^2}{6h} \right)$

$$= m b \sigma_b \left(\frac{t^2}{6mh} \right)$$

$(Y \propto z)$

Lewis eqn
(Bending strength eqn)

$$P_t = m b \sigma_b Y$$

$f(z) \rightarrow Y = \text{Lewis form factor}$

\rightarrow pinion is weaker \rightarrow pinion will be used for design (material is same)

QUIZ

Lewis Equation

Which of the following is NOT an assumption made while deriving Lewis equation?

1. Radial force is included by its proportion to the tangential force
2. Stress concentration effect is neglected ✓
3. Only one pair of tooth take full load at a time ✓

QUIZ

Lewis Factor (Y)

Lewis factor (Y) _____ with number of teeth.

1. Increases ✓
2. Decreases

QUIZ

Allowable bending stress

depends on

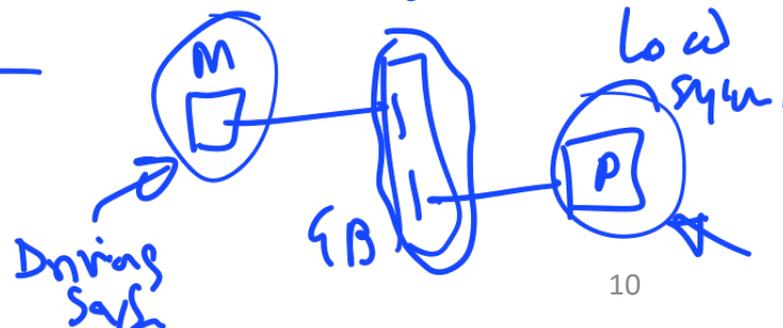
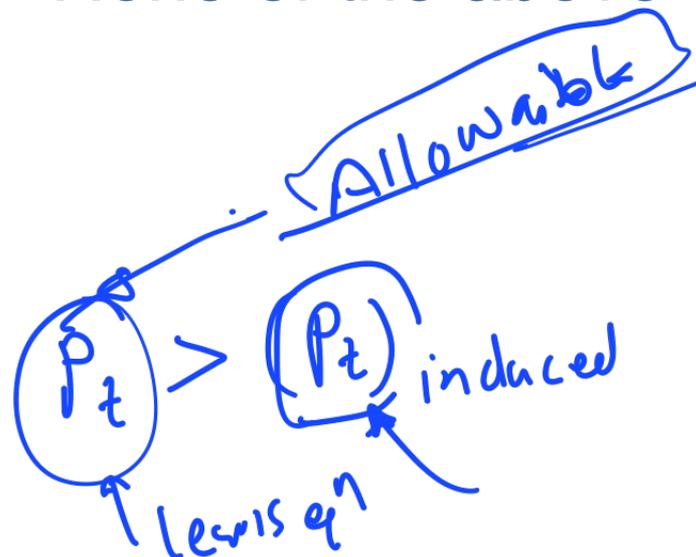
Allowable bending stress in Lewis equation is _____

1. Tensile yield strength
- ✓ 2. Tensile ultimate strength
3. None of the above

$$P_t = m b \sigma_b Y$$

allowable bending stress

$$\left(\frac{UTS}{3} \right)$$



$$P_{eff} = C_s \times P_t$$

→ service factor
← theoretical tang. force

Effective load on gear tooth

→ induced load on gear tooth

- Effective load on gear tooth (Velocity factor method)

ALTERNATIVELY

- Effective load as per Buckingham equation

Dynamic load on gear tooth

- inaccuracies in manufacturing gear tooth
- errors in tooth spacing
- misalignment between beams
- elasticity of parts
- inertia of parts
↳ mass $m \cdot I$.

Velocity factor method

Buckingham load method